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RADC-TDR-64-317

AN ANALYSIS OF THE SCATTERING MATRIX MEASUREMENT CAPABILITIES OF A GROUND PLANE RADAR CROSS SECTION RANGE

TECHNICAL DOCUMENTARY REPORT NO. RADC-TDR-64-317

June 1964

Space Surveillance and Instrumentation Branch
Rome Air Development Center
Research and Technology Division
Air Force Systems Command
Griffiss Air Force Base, New York

Project No. 6503

(Prepared by General Dynamics/Fort Worth
A Division of General Dynamics Corporation
Under Contract No. AF30(602)-2831)

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FOREWORD

In order to meet the need for a National Radar Reflectivity Range, Rome Air Development Center (RADC) awarded a development contract on 29 June 1962 to General Dynamics/Fort Worth (GD/FW) to design, fabricate, and develop the Radar Target Scatter Site (Project RAT SCAT) on the Alkali Flats, Holloman AFB, New Mexico, (Contract AF30(602)-2831). The operational RAT SCAT Site was delivered to the Air Force on 30 June 1964.

The RAT SCAT facility was developed for full-scale radar cross section measurements. In the pursuit of this development, an R&D Program was undertaken to provide for the specific needs of Project RAT SCAT as requirements appeared in the implementation of the function of the Site. A significant portion of this work was subcontracted. Emphasis was placed on those areas thought to be most promising in achieving measurement objectives. The presentation of the results of the R&D Program is covered in eight reports which were prepared as RADC Technical Documentary Reports.

This report (GD/FW Report No. FZE-222-4) is No. 4 in the series; it contains a description of the Phase I task accomplished by Cornell Aeronautical Laboratory, Inc., and reported in their Report No. UB-1806-P-1, dated January, 1964, which was prepared by Mr. R. A. Ross. The scattering matrix measurements were performed by General Dynamics/Fort Worth and are documented along with the results of the circular polarization tests in RADC-TDR-64-380, entitled Experimental Results of Circular Polarization and Scattering Matrix Measurements, dated June, 1964.

The contents of this report and the abstract are unclassified.

RADC-TDR-64-317
June 1964

ABSTRACT

The concept of the polarization scattering matrix is presented in this report. Measurement techniques leading to the determination of the scattering matrix on a radar range are discussed. The polarization dependence of measurements on a ground plane range are noted. Matrix-determination techniques are examined in the context of ground-plane-range operation. The utility of performing scattering matrix measurements on the RAT SCAT range is discussed. Finally a specific measurement technique is chosen for the scattering matrix solution; consideration of operational ease and measurement accuracy forms the basis of this choice. This report is No. 4 in a series of 8 RAT SCAT Research and Development Program Reports.

PUBLICATION REVIEW

This report has been reviewed and is approved. For further technical information on this project, contact

William P. Bond, Capt

Approved: DONALD M. MONTANA
Program Directors' Office
Space Surveillance and Instrumentation Branch

Approved:

[illegible]
Colonel, USAF
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SECTION 1

INTRODUCTION

The concept of a scattering matrix arises in the field of radar cross sections when target characteristics are to be expressed as functions independent of radar characteristics. The particular scattering matrix employed in this report can be termed the polarization scattering matrix of the target. Since the quantity "scattering matrix" has had different meanings to various investigators, its origin and present implicationw will be discussed.

Heisenberg introduced the concept of a scattering matrix to allow study of scattering processes without employing a particular interaction. Sinclair was the first to apply such a matrix to the specific field of radar cross sections (Reference 1). He considered the modifications required in the radar range equation to permit its application to the general case of an arbitrary target with arbitrary polarization of the transmitting and receiving antennas in bistatic as well as monostatic modes of operation. This work led to matrix formulations of antenna polarizations and target scattering characteristics. His scattering matrix completely characterizes the scattering properties of the target. The elements of this matrix are functions of radar frequency, transmitter-receiver configuration, and target aspect.

Another method of representing the scattering properties of a body is the scattering matrix proposed by Tsu (Reference 2). Tsu determines the elements of the matrix by a multipole expansion of the expression for scattering from the body, and the matrix contains an infinity of elements; his matrix contains all of the scattering properties of the body and permits the determination of scattering at any angle for any radar frequency. For a small body of the order of a wavelength in size only a few elements of the matrix are required, because the scattering is well represented by the dipole or quadrupole approximation. Even in these cases, the problem is quite complicated if the body is not axially symmetric. For large bodies, determination of the values of the matrix elements becomes nearly impossible. Tsu's scattering matrix is not given further consideration in this report.

A third alternative discussed by Graves (Reference 3) replaces the notion of the usual scattering matrix (a voltage quantity) with that of a power scattering matrix. Treatment of the power scattering matrix is temporarily withheld: a more meaningful discussion results when the power scattering matrix is presented in the context of Section 3.

More recent work with target scattering matrices has tended toward the use of a simplified form which does not completely characterize the scattering properties of the target. (See References 4, 5, 6, and 7.) The simplified matrix applies for one radar frequency, one transmitter-receiver configuration (usually monostatic) and a single orientation of the target. Knowledge of the simplified scattering matrix completely determines the polarization properties of the target at this frequency, transmitter-receiver geometry, and target aspect. In the remainder of this report the term "scattering matrix" is used in the simplified sense, i.e., the polarization scattering matrix, unless otherwise noted.

It is usual to measure the polarization scattering matrix at various target aspects in order to obtain more complete definition of target scattering characteristics. Further information on target scattering can be obtained by varying the radar frequency. A complete characterization of target scattering involves additional measurements at all bistatic transmitter-receiver combinations.

SECTION 2

SCATTERING MATRIX FORMULATIONS

Before the detailed discussion of scattering matrices is begun, the terms and notations to be used must be defined. The notation is that used by Deschamps (Reference 8). A right-handed coordinate system with radar transmission in the +Z direction is assumed (Figure 1).

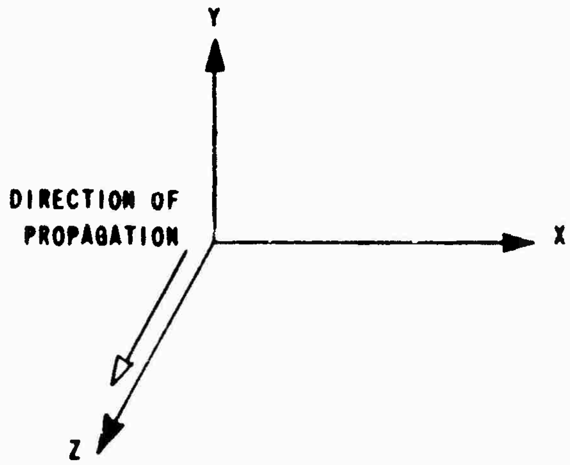


Fig. 1 COORDINATE SYSTEM

An elliptically polarized wave can be represented by a pair of orthogonal linearly-polarized waves:

$$E_x = E_o \cos \gamma e^{j(\omega t - kz)} \tag{1a}$$

$$E_y = E_o \sin \gamma e^{j(\omega t - kz + \delta)} \tag{1b}$$

Here E_o is an amplitude factor, γ is an angle ($0 \leq \gamma \leq 90°$) representing the orientation of the linear polarization that results if δ is zero, δ is a phase angle ranging from 0 to 360°, ω is the radian frequency, and k($=\omega/c$) is the wave number. Any wave polarization is thus specified when γ, δ, and the direction of propagation are known. (The fourth parameter, E_O, specifies the amplitude of the wave and is unrelated to the type of elliptical polarization.)

The sign of δ depends upon the sense of the wave, i.e., upon whether the tip of the E vector rotates clockwise or counterclockwise, and upon the direction of propagation. Consider a wave propagating in the +Z direction as shown in Figure 1. From Equations 1a and 1b, for $Z = 0$ and $\delta > 0$, the rotation is counterclockwise looking in the direction of propagation and the wave is assigned left-sense polarization.* Conversely, if $\delta < 0$, the wave is of right-sense polarization.

To permit discussion of the polarization properties of targets in terms of scattering matrices, it is first necessary to define matrices giving the polarization properties of antennas. Consider a transmitting antenna; this antenna can be represented by the expression

$$\hat{q} = \begin{bmatrix} \cos \gamma_t \\ \sin \gamma_t \, e^{j\delta_t} \end{bmatrix} \tag{2}$$

where $\hat{q}$ is a unit column matrix defining the polarization of the transmitted wave. If antenna $\hat{q}$ is excited by a unit voltage it produces a wave

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} \cos \gamma_t \\ \sin \gamma_t \, e^{j\delta_t} \end{bmatrix} \tag{3}$$

which is the wave of Equation 1. A constant relating field strength to terminal voltage is dropped from this and succeeding equations, because only the (normalized) polarization properties of the antennas are desired. Next, consider an antenna represented by the matrix $\hat{p}$:

$$\hat{p} = \begin{bmatrix} \cos \gamma_r \\ \sin \gamma_r \, e^{j\delta_r} \end{bmatrix} \tag{4}$$

If the second antenna is used to receive the wave radiated from the first, the voltage generated at the terminals of the second antenna is given by

$$V = \hat{p}_t^{*} \hat{q} \tag{5}$$

*This definition of sense of polarization is that established by the IRE (1942) Standard on Radio Wave Propagation, Definitions and Terms.

where the asterisk indicates the complex conjugate and the subscript t indicates the transposed matrix. In general, the $\hat{p}$ matrix is written as a row matrix to avoid the need for the transpose notation. The conjugate is required because the receiving antenna operates on an approaching, rather than a receding, wave. If two identical antennas are used, one to receive the wave transmitted by the other, the received (normalized) voltage is unity. The antenna having $\gamma_r = \pi/2 - \gamma_t$ and $\delta_r = \pi + \delta_t$ can also be used to receive the wave transmitted by ; in this case, the received voltage is zero. This latter antenna is said to be of a polarization orthogonal to that produced by the first antenna.

The polarization properties of a radar target must also be represented by a matrix if the voltage (or, equivalently, the power) received from the target when it is illuminated by antenna $\hat{q}$ and observed by antenna $\hat{p}$ is to be determined. This matrix expresses the field strengths of orthogonal components of the scattered electric (or magnetic) field in terms of the corresponding components of the incident electric (or magnetic) field:*

$$\begin{bmatrix} E_1^r \\ E_2^r \end{bmatrix} = \begin{bmatrix} S \end{bmatrix} \begin{bmatrix} E_1^i \\ E_2^i \end{bmatrix} \tag{6}$$

where the superscripts i and r indicate incident and reflected fields respectively. The two components, $E_1^{r,i}$ and $E_2^{r,i}$, may be related to any orthogonal system of bases; vertical and horizontal or right- and left-circular polarizations are often chosen.

Strictly speaking, tensor rather than matrix notation should to describe radar scattering in Equation 6. However, it is usual to assume that the intervening medium is homogeneous and isotropic, and there are linear relationships between the field quantities at every point, whatever the incident field. Under this assumption, no advantage would accrue from tensor notation and the scattering matrix, a more familiar concept, is used.

By properly choosing coordinates to take advantage of the transverse character of the far field, viz., by taking a coordinate in the direction from body to receiving antenna, the general

*It is implicitly assumed that the far zone electric and magnetic fields are proportional -- that one antenna is in the free-space field of the other.

3 x 3 scattering matrix may be expressed as a 2 x 2 matrix. The far field assumption requires the distance between the receiver and body to be large compared to the wavelength and to the dimensions of the scattering body. In this situation, the radial component of the scattered field, which behaves as $1/r^2$, can be neglected in comparison to the transverse components of the scattered field which are $1/r$ dependent.

The assumptions of linearity and far field lead to the following form for the scattering matrix of an arbitrary body.

$$[S] = \begin{bmatrix} a_{11} e^{j\phi_{11}} & a_{12} e^{j\phi_{12}} \\ a_{21} e^{j\phi_{21}} & a_{22} e^{j\phi_{22}} \end{bmatrix} \tag{7}$$

Here a_{ij} represents the real part or magnitude of the scattering-matrix elements, and ϕ_{ij} denotes the associated phase angle. The scattering matrix is symmetrical ($a_{ij} = a_{ji}$; $\phi_{ij} = \phi_{ji}$) in at least one important case: backscattering from an arbitrary scattering body, consistent with the two assumptions stated previous to Equation 7. Removing the factor $e^{j\phi_{11}}$ from the matrix and writing $\theta_s = \phi_{12} - \phi_{11}$, $\psi_s = \phi_{22} - \phi_{11}$, one can write the symmetrical matrix in the form

$$[S] = e^{j\phi_{11}} \begin{bmatrix} a_{11} & a_{12} e^{j\theta_s} \\ a_{12} e^{j\theta_s} & a_{22} e^{j\psi_s} \end{bmatrix} \tag{8}$$

where ϕ_{11} may be ignored in the present discussion since it is a function of the distance from the radar to the target. The symmetrical matrix form is used in all subsequent analysis. The element magnitudes a_{11} , a_{12} and a_{22} are positive real quantities and the element phase angles θ_s and ψ_s are unrestricted in value (although they must be real).

Consider a body having a scattering matrix $[S]$ illuminated by an antenna characterized by matrix $\hat{q}$ and observed by an antenna characterized by matrix $\hat{p}$. The normalized voltage at the receiver is

$$V = [\hat{p}] [S] [\hat{q}] \tag{9}$$

Here the matrix $\hat{p}$ is the transpose of the matrix given in Equation 4. The scattering matrix $[S]$ is not necessarily a unit matrix, but the range dependence of the elements has been removed (each element should contain a $\frac{1}{r}$ factor). The effective radar cross section of the target for the antenna system used is here defined by the equation

$$\sigma = VV^* = |V|^2 = |\hat{p} S \hat{q}|^2 \tag{10}$$

Note that in Equation 10 the conjugate of the matrix for the receiving antenna is not used; this change from Equation 5 is required by the reversal in direction of propagation of the radar wave effected by the scattering body. Antenna matrices $\hat{p}$ and $\hat{q}$ each represent the transmission properties of the antenna in question.

There are many formulas that, while useful in various systems analyses, are not required in the specific analyses discussed in later section of this report. Appendix A lists some of these formulas for the interested reader.

SECTION 3

DETERMINATION OF THE SCATTERING MATRIX

The following discussion considers the determination of a target scattering matrix and points out the complexity of such an operation. The analysis treats the symmetrical scattering matrix introduced in Section 2,

$$[S] = \begin{bmatrix} a_{11} & a_{12}e^{j\theta_s} \\ a_{12}e^{j\theta_s} & a_{22}e^{j\psi_s} \end{bmatrix} \tag{11}$$

where the absolute phase of $[S]$ is neglected. Knowledge of $[S]$ completely defines the target polarization characteristics for the particualr frequency, radar bistatic angle, and target aspect angle involved: the cross section of the target may be calculated for any desired transmitting and receiving antenna combination. The complexity of the matrix derives from five independent variables, of which three (a_{11} , a_{12} , a_{22}) are always positive and two (θ_s , ψ_s) are signed quantities.

It is to be expected that the determination of $[S]$ involves complexity either in the required number of measurements or in the measuring equipment. Solutions to the scattering matrix can be classed according to three broad techniques depending on equipment capabilities:

1. Amplitude and absolute-phase measurements
2. Amplitude and relative-phase measurements
3. Amplitude-only measurements.

The above methods of determination of the scattering matrix are listed in order of decreasing equipment complexity. Each technique will be examined in terms of the minimum number of radar transmissions and receptions required for the $[S]$ solution. Where phase measurements are implied in the equipment description, the equipment problems in phase measurement are not given weight when techniques are compared. In this sense, the three matrix solution techniques are compared only on the basis of the required numbers of radar transmissions and cross-section receptions.

AMPLITUDE AND ABSOLUTE-PHASE MEASUREMENTS

The scattering matrix can be completely determined by transmitting only two linear polarizations and receiving three linear polarizations if absolute phase as well as amplitude measurements are made:

Transmissions	Receptions	Measured Parameters
Vertical	Vertical	a_{22}, ϕ_{22}
	Horizontal	a_{12}, ϕ_{12}
Horizontal	Horizontal	a_{11}, ϕ_{11}

The measured parameters are a_{ij} and ϕ_{ij}. (If cross sections are recorded, the quantity a_{ij}^2 has been obtained.) The matrix phase angles θ_s and ψ_s are obtained from the relations $\phi_{12}-\phi_{11}=\theta_s$ and $\phi_{22}-\phi_{11}=\psi_s$ respectively. Because of the difficulty in obtaining absolute phase measurements, no scattering matrix solutions have utilized this approach. However, such a facility is presently being developed at Lockheed.*

AMPLITUDE AND RELATIVE-PHASE MEASUREMENTS

Relative phase measurements, which are simpler to make than absolute phase measurements, permit the determination of the scattering matrix from the results of two linear transmissions and four linear receptions.

	Transmissions	Receptions	Measured Parameters
(1)	Vertical	Vertical) Horizontal) simul.	$\left.\begin{matrix} a_{22} \\ a_{12} \end{matrix}\right\} \phi_{22}-\phi_{12}=\psi_s-\theta_s$

If $a_{12} \neq 0$ proceed with following measurements

	Transmissions	Receptions	Measured Parameters
(2)	Horizontal	Horizontal) Vertical) simul.	$\left.\begin{matrix} a_{11} \\ a_{12} \end{matrix}\right\} \phi_{12}-\phi_{11}=\theta_s$

If $a_{12}=0$, replace 2 by

	Transmissions	Receptions	Measured Parameters
(3)	$\pi/4$	Horizontal) Vertical) simul.	$\left.\begin{matrix} a_{11}/\sqrt{2} \\ a_{22}/\sqrt{2} \end{matrix}\right\} \phi_{22}-\phi_{11}=\psi_s$

*Personal conversation with Paul Kresge of the Operations Research Branch, Lockheed, Georgia.

Simultaneous reception of two receiver channels is required in the extraction of relative phase. For bodies which produce $a_{12} = 0$, measurement set 2 fails to provide the matrix phase angle ψ_s . In this situation, measurement set 2 would be superseded by measurement set 3, in which $\pi/4$ denotes a linear polarization halfway between vertical and horizontal. If such a dynamic procedure proved untenable, the present matrix solution technique then would require three transmissions and six receptions.

The technique of measuring amplitude and relative phase has been used to obtain scattering matrices from cross-section range facilities (Reference 6).

AMPLITUDE-ONLY MEASUREMENTS

When amplitude alone is measured, cross-section measurements do not lead to a direct determination of the scattering matrix, but, rather, provide the coefficients of equations from which the matrix elements are deduced. Five independent magnitude-determining measurements plus two sign-determining measurements must be made. In all, therefore, seven amplitude measurements are needed to completely determine the target polarization characteristics.

Kennaugh suggest seven measurements using the same transmitter and receiver polarizations (Reference 4). Only one of these measurements need be with other than linear polarization. This use of the amplitude-only technique has been employed in the cross-section range determination of the scattering matrices of targets each having a plane of symmetry containing the radar line of sight (Reference 7). For the symmetrical target, $x_{12} = 0$ and three measurements using the same transmitter and receiver polarizations provide the scattering matrix.

An investigation of the reduction in the required number of transmissions indicates that at least three linear transmissions are necessary to find the general (symmetrical) scattering matrix from amplitude measurements only. A combination of two opposite-sense circular transmissions plus one linear transmission also can be used to find the matrix. In the former case, ten receptions are required; in the latter case, nine. Examples of the two minimal-transmission methods using the amplitude-only technique are given below.

1. **Method Based Upon Linear Transmissions Only**

	Transmissions	Receptions	Measured Parameters	Calculated Parameters
(1)	Vertical	Vertical	a_{22}	
		Horizontal	a_{12}	

If $a_{12} \neq 0$ in 1, proceed with following measurements.

(2)	Vertical	$\pi/4$		$\cos(\theta_s - \psi_s)$
		Rt. Circ.		$\sin(\theta_s - \psi_s)$
	Horizontal	Horizontal	a_{11}	
		Rt. Cirv. $\pi/4$		$\sin \theta_s$ $\cos \theta_s$

If $a_{12} = 0$ in 1 replace 2 by

(3)	Horizontal	Horizontal	a_{11}	
	$\pi/4$	$\pi/4$		$\cos \psi_s$
		Rt. Circ.		$\sin \psi_s$

Because phase measurements are not taken, the matrix phase angles are listed under the column headed "Calculated Parameters". Again, measurement set 3 must be included as an alternative to 2 to provide ψ_s when $a_{12} = 0$.

2. Methods Using Circularly Polarized Antennas

The scattering matrix cannot be determined from amplitude measurements alone if only circular-polarization antennas are used for transmission. The following approach requires one additional linear-polarization transmission.

Transmissions	Receptions	Calculated Parameters
Rt. Circ.	Vertical Horizontal L. Circ.	All measurements result in equations relating three or more elements of the scattering matrix
L. Circ.	Vertical Horizontal	
$\pi/4$	Vertical Horizontal Rt. Circ. $(-\pi/4)$	(necessary when a_{12} =0)

In the above statement of antenna polarizations, reception at $(-\pi/4)$ represents a linear antenna orthogonal to the $\pi/4$ transmitting antenna.

COMPARISON OF MATRIX-DETERMINATION TECHNIQUES BASED UPON RANGE MEASUREMENT ACCURACY

Techniques for the determination of a target scattering matrix have already been outlined. In an analysis of range measurement errors contained in Appendix B, it is shown that certain matrix forms can produce large inaccuracies in calculated cross sections for specific antenna polarization combinations. It is concluded that these large inaccuracies stem from errors in the estimation of matrix phase angles. Matrix measurement techniques will now be compared in terms of the accuracy with which matrix phase angles are determined.

Consider first the determination of a target scattering matrix using the amplitude-only measurement technique. Here the matrix phase angles are derived from a set of simultaneous equations relating measured cross sections: errors in matrix phase angles result from errors in measured cross sections. It is evident that nominal ($\pm$ 1 db) errors in measured cross sections can produce large errors in matrix phase angle estimates. (Indeed, there will be times when the derivation will equate a sine or a cosine of the matrix phase angle to a magnitude greater than one!) It is recommended that subsequent effort investigate the errors in matrix phase angles resulting from errors in measured cross sections. This work would parallel the error analysis presented in Appendix B.

Direct measurement of matrix phase angles will probably result in more accurate data than the indirect method treated above. In particular, measurement of relative phase should provide the best estimate of matrix phase angles.

The susceptibility of certain scattering matrix computations to errors in the determination of matrix phase angles suggests the desirability of formulating a new scattering matrix defined by element magnitudes only. Such a scattering matrix is treated in Reference 3. Graves has shown that the power scattering matrix of a target (involving real elements only) is completely specified by transmitting at horizontal, vertical, $\pi/4$ plane polarization, and circular polarization and measuring the total power back-scattered after each transmission. (The total power measurement is easily obtained by reception on two orthogonal linearly polarized antennas.) However, the power scattering matrix does <u>not</u> provide cross sections for arbitrary combinations of transmitting and receiving antenna polarizations; what it does is provide the total back-scattered power for any transmitting antenna polarization. For this reason, the power scattering matrix is not considered an acceptable replacement for the conventional voltage scattering matrix.

SECTION 4

CROSS-SECTION MEASUREMENTS ON A GROUND-PLANE RANGE

THE GROUND-PLANE CONCEPT

The ground-plane range, used successfully for antenna pattern measurements, has been proposed as a technique for radar cross-section measurement. In such a ground-plane range the ground is made flat and smooth with respect to λ , the operating wavelength. The antenna and scatterer are located in that region of space where the direct ray and the reflected ray produce a maximum (Figure 2).

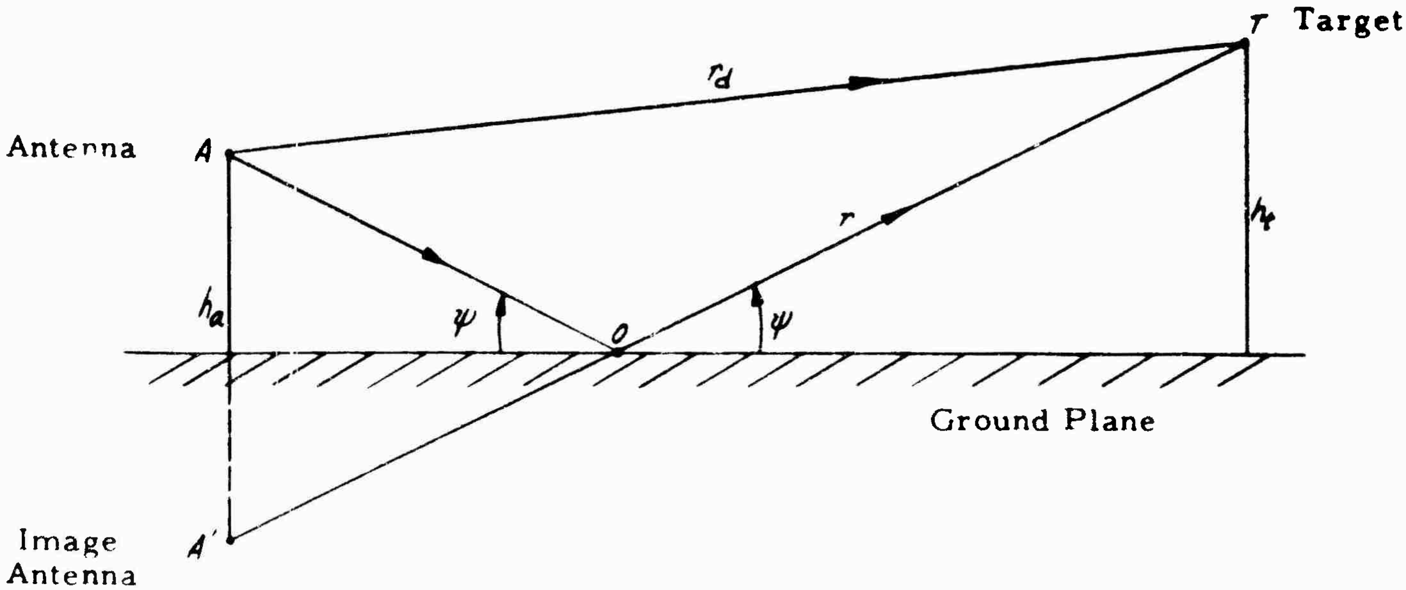


Fig. 2 GROUND PLANE INTERACTION

In the above figure the height of the radar antenna A above the ground plane is h_a, the height of the target T is h_t. Both h_a and h_t are several wavelengths or more, and r_d, the separation between A and T, is so large that the field at the target falls off as $1/r_d$. The image of the antenna A is at A', a distance h_a below the reflecting ground plane. The length of the ray from A' to the target T is r. Reflection of the energy transmitted at A occurs at the ground plane position 0 at the grazing angle ψ .

The coefficient of reflection is $\mathcal{R}$, and the phase shift at reflection is ψ . The electric field strength due to the combined direct and reflected waves at the target may be written

$$E = \frac{E_1}{r_d} e^{j(\omega t + k r_d)} + \frac{\mathcal{R} E_1}{r} e^{j(\omega t + k_r + \psi)}$$

$$= E_1 e^{j(\omega t + k r_d)} \left[\frac{1}{r_d} + \frac{\mathcal{R}}{r} e^{j(kr - kr_d + \psi)} \right] \tag{12}$$

where the difference $(r-r_d)$ is important in the exponent and $r_d \sim r$ is valid in the denominator. Substituting $\Delta = (2\pi/\lambda)(r-r_d)$, we have

$$E = \left[1 + \mathcal{R} e^{j(\psi + \Delta)} \right] \frac{E_1}{r} e^{j(\omega t + k r_d)} \tag{13}$$

Range geometry is chosen so that the target T is located in the maximum of the first lobe. The amplitude pattern implied in Equation 13 is of the form

$$\sqrt{1 + \mathcal{R}^2 + 2\mathcal{R} \cos(\psi + \Delta)} \tag{14}$$

The polarization dependence of fields above the ground plane is evident in the expression of the reflection coefficient in terms of the ground constants (Reference 11). For horizontal polarization of the transmitting antenna A (E field parallel to the ground plane)

$$\mathcal{R}_H e^{j\psi_H} = \frac{\sin\psi - \sqrt{(k_1/k_0)^2 - \cos^2\psi}}{\sin\psi + \sqrt{(k_1/k_0)^2 - \cos^2\psi}} \tag{15}$$

Similarly for vertical polarization

$$\mathcal{R}_V e^{j\psi_V} = \frac{(k_1/k_0)^2 \sin\psi - \sqrt{(k_1/k_0)^2 - \cos^2\psi}}{(k_1/k_0)^2 \sin\psi + \sqrt{(k_1/k_0)^2 - \cos^2\psi}} \tag{16}$$

where ψ is the grazing angle and $(k_1/k_0)^2$ is the complex dielectric constant of the ground. In the following analysis it will be convenient to use the notation

$$\Gamma_H e^{j\rho_H} = \left[1 + \mathcal{R}_H e^{j(\psi_H + \Delta)} \right]$$

$$\Gamma_V e^{j\rho_V} = \left[1 + \mathcal{R}_V e^{j(\psi_V + \Delta)} \right] \tag{17}$$

In the ideal case $\Gamma e^{j\rho}$ would be independent of polarization and real with magnitude 2. In actual practice Γ approximates the value 2, ρ is some small angle, and the polarization dependence must be retained.

THE WELL-BEHAVED GROUND-PLANE RANGE

The fundamental requirement for the measurement of cross section is that the phase and amplitude structure of the field encompassing the target approach the uniformity associated with the long range at which an operational target would be observed. While this requirement applies to ranges in general, the ground plane range introduces additional constraints. A ground plane range will be called well behaved if it satisfies the following conditions:

1. Vertical Coverage Diagram Overlap

   There must exist a common volume (the target location) of overlap at the first-lobe maximums for both vertical and horizontal polarization field patterns. It is the field in this common volume that should satisfy the uniformity requirement. The resultant amplitude and phase of the field for horizontal polarization need not be equal to those for vertical polarization. (In practice, the horizontal polarization field strength at the first-lobe maximum is greater than the vertical-polarization field strength.) The overlap condition must be satisfied if cross section measurements employing polarizations other than the principal polarizations are anticipated.

2. Constancy of Ground Parameters

   The electrical properties of the ground are influenced by weather and seasonal growth. The physical configuration of the ground plane is subject to winds, etc. It is necessary that ground parameters which affect the range field patterns remain essentially constant over the time interval between range calibrations. Only in this situation can compensation for ground plane effects be considered meaningful.

The well-behaved ground-plane range is defined to be a ground plane range producing cross-section measurements suitable for compensation, no matter what polarization combinations are involved.

CROSS-SECTION MEASUREMENTS ON A GROUND-PLANE RANGE

Consider the measurement of cross section over a well behaved ground-plane range. Writing the ground-plane polarization dependence in matrix form we have

$$T = \begin{bmatrix} \Gamma_H e^{j\rho_H} & 0 \\ 0 & \Gamma_V e^{j\rho_V} \end{bmatrix} \tag{18}$$

The transmitter polarization $\hat{q}$ will have undergone the transformation T upon reaching the target. Similarly, the scattered wave $[S][T]\hat{q}$ will experience the same transformation on the way to the receiver. The scattering-matrix formulation for cross section measured over the ground plane becomes

$$\begin{aligned}(\sigma_{\hat{p}\hat{q}})_{g.p.} &= \left| [\hat{p}][T][S][T][\hat{q}] \right|^2 \\ &= \left| \Gamma_H^2 e^{j2\rho_H} a_{11} \cos\gamma_t \cos\gamma_r + \Gamma_V^2 e^{j2\rho_V} a_{22} e^{j\theta_2} \sin\gamma_t \sin\gamma_r e^{j(\delta_t+\delta_r)} \right. \\ &\quad \left. + \Gamma_H \Gamma_V e^{j(\rho_H+\rho_V)} a_{12} e^{j\theta_3} \left(\cos\gamma_t \sin\gamma_r e^{j\delta_r} + \sin\gamma_t \cos\gamma_r e^{j\delta_t}\right) \right|^2\end{aligned} \tag{19}$$

Next let us examine calibration techniques which cancel the effects of the ground plane transformation matrix T. Any measured cross section $(\sigma_{\hat{p}\hat{q}})_{g.p.}$ which involves only one of the three terms indicated in Equation 19 may be made independent of the transformation matrix T through normal calibration procedures.

(It is usual to measure radar cross section by comparing the power received from the target with the power received from a calibration sphere which has a known radar cross section.) Cross section measurements of an arbitrary target using the two principal polarizations (horizontal-horizontal and vertical-vertical antenna polarization combinations) are free from the disturbing influence of ground-plane polarization dependence when referenced to a calibration sphere. If one transmits and receives horizontal linear polarizations ($\gamma_t = \gamma_r = 0, \delta_t = \delta_r = 0$),

$$\left(\frac{\sigma_{target}}{\sigma_{sphere}}\right)_{g.p.} = \left|\frac{a_{11\,target}\,\Gamma_H^2 e^{2j\rho_H}}{a_{11\,sphere}\,\Gamma_H^2 e^{2j\rho_H}}\right|^2 = \frac{\sigma_{target}}{\sigma_{sphere}} \tag{20}$$

Similarly, in the vertical polarization case ($\gamma_t = \gamma_r = \pi/2, \delta_t = \delta_r = 0$)

$$\left(\frac{\sigma_{target}}{\sigma_{sphere}}\right)_{g.p.} = \left|\frac{a_{22}\, e^{j\theta_3}_{target}\,\Gamma_V^2 e^{2j\rho_V}}{a_{22}\, e^{j\theta_3}_{sphere}\,\Gamma_V^2 e^{2j\rho_V}}\right|^2 = \frac{\sigma_{target}}{\sigma_{sphere}} \tag{21}$$

A third cross-section measurement which results in use of a single term of Equation 19 is associated with crossed-vertical (or equivalently, crossed-horizontal) linear polarization measurements. Then $\gamma_t = \pi/2, \gamma_r = 0, \delta_t = \delta_r = 0$ (or $\gamma_t = 0, \gamma_t = \pi/2, \delta_t = \delta_r = 0$) and we have

$$\left(\frac{\sigma_{target}}{\sqrt{\sigma_{sphere_{HH}}\,\sigma_{sphere_{VV}}}}\right)_{g.p.} = \left|\frac{a_{12\,target}\,\Gamma_H \Gamma_V e^{j\theta_2}}{\sqrt{a_{11\,sphere}\,\Gamma_H^2 \times a_{22\,sphere}\,\Gamma_V^2}}\right|^2 = \frac{\sigma_{target}}{\sigma_{sphere}} \tag{22}$$

Notice that reference sphere cross sections at two principal polarizations are required in order to calculate the desired cross section of the target. It is concluded that the only cases where

T cancellation can be obtained by referencing cross section to that of a calibration sphere demand measurement at principal polarizations only.

Cross-section measurements employing antenna polarizations other than those discussed above require formal and explicit compensation to eliminate the ground plane transformation matrix T. The obvious compensation procedure involves use of corrected antenna polarizations $\hat{q}_c$ and $\hat{p}_c$, where

$$\hat{q}_c = [\hat{q}][T]^{-1} \quad \text{and} \quad p_c = [\hat{p}][T]^{-1} \tag{23}$$

Then

$$(\sigma_{\hat{p}_c \hat{q}_c})_{g.p.} = \left| [\hat{p}_c][T][S][T][\hat{q}_c] \right|^2 = \sigma_{\hat{p}\hat{q}} \tag{24}$$

Data compensation during the measurement operation requires either accurate measurement of the ground plane transformation matrix (which is a function of operating frequency) or the inclusion of new reference targets in the calibration procedure. One such reference target is a dipole oriented at 45 degrees to the principal polarizations.

SECTION 5

SCATTERING-MATRIX MEASUREMENTS ON A GROUND-PLANE RANGE

This section is concerned with the determination of the scattering matrix of an arbitrary target by measurements on a well-behaved, ground-plane range. It has just been shown that two cross sections, σ_{HH} and σ_{VV}, remain unperturbed by the ground plane polarization dependence, provided they are measured relative to a reference sphere. However, the investigation of techniques for the determination of the target scattering matrix revealed that cross sections at polarization combinations other than the principal polarizations must be measured. Thus it is expected that the set of measurements associated with the scattering matrix solution will require some form of compensation to offset the polarization dependence of the ground plane. The relative ease with which compensation can be effected is illustrated by reconsideration of scattering-matrix techniques in the light of ground-plane range operation. Scattering-matrix measurements on a ground plane range provide data related to the apparent target matrix S', where

$$S' = \left[T\right]\left[S\right]\left[T\right]$$

$$= e^{j\xi}\begin{bmatrix} \Gamma_H^2 e^{j2\rho_H} a_{11} e^{j\phi_{11}} & \Gamma_H \Gamma_V e^{j(\rho_H+\rho_V)} a_{12} e^{j\phi_{12}} \\ \Gamma_H \Gamma_V e^{j(\rho_H+\rho_V)} a_{12} e^{j\phi_{12}} & \Gamma_V^2 e^{j2\rho_V} a_{22} e^{j\phi_{22}} \end{bmatrix} \tag{25}$$

$$\equiv \begin{bmatrix} \Gamma_H^2 a_{11} & \Gamma_V \Gamma_H a_{12} e^{j(\theta_S+\rho_V-\rho_H)} \\ \Gamma_V \Gamma_H a_{12} e^{j(\theta_S+\rho_V-\rho_H)} & \Gamma_V^2 a_{22} e^{j(\varphi_S+2(\rho_V-\rho_H))} \end{bmatrix} \tag{26}$$

Cross-section measurements are calibrated in the usual manner using a sphere of known cross section as a reference. However, on a ground-plane range the reference sphere cross sections $\sigma_{HH_{SP}}$ and $\sigma_{VV_{SP}}$ are not equal, even though [illegible] = [illegible], because of the polarization dependence. For this reason the reference cross sections will be included in the measured parameter column.

AMPLITUDE AND ABSOLUTE-PHASE MEASUREMENTS

Equation 25 is used to generate measurement data obtained from a ground plane range. Compare the following results with those given in Section 3 under Amplitude and Absolute-Phase Measurements.

Transmissions	Receptions	Measured Parameters
Vertical	Vertical	$\frac{(a_{22\,Target}\,\Gamma_V^2)}{(a_{22\,Sphere}\,\Gamma_V^2)}$; $(2\rho_V + \phi_{22} + \xi)$
	Horizontal	$\frac{(a_{12\,Target}\,\Gamma_V\,\Gamma_H)}{(a_{22\,Sphere}\,\Gamma_V^2)}$; $(\rho_V + \rho_H + \phi_{12} + \Xi)$
Horizontal	Horizontal	$\frac{(a_{11\,Target}\,\Gamma_H^2)}{(a_{11\,Sphere}\,\Gamma_H^2)}$; $(2\rho_H + \phi_{11} + \xi)$

The first and third cross-section measurements provide a_{11} and a_{22}, elements of the true target matrix, because of the cancellation of ground-plane polarization dependence through the usual cross-section calibration. The second cross-section measurement gives the quantity

$$\frac{(a_{12\,Target}\,\Gamma_V\,\Gamma_H)}{(a_{22\,Sphere}\,\Gamma_H^2)}.$$

If the second cross-section measurement is referenced to the horizontal-horiaontal polarization sphere cross section, we have

$(a_{12\,Target}\,\Gamma_V\,\Gamma_H)/(a_{11\,Sphere}\,\Gamma_H^2)$. Now multiply these two quantities and get

$$\frac{(a_{12\,target}\,\Gamma_V\,\Gamma_H)}{(a_{22\,sphere}\,\Gamma_V^2)} \cdot \frac{(a_{12\,target}\,\Gamma_V\,\Gamma_H)}{(a_{11\,sphere}\,\Gamma_H^2)} = \frac{a_{12\,target}^2}{a_{11\,sphere}\,a_{22\,sphere}} = \frac{a_{12\,target}^2}{a_{11\,sphere}^2} \qquad (27)$$

The element a_{12} has been obtained independent of ground-plane polarization effects.

Working with the absolute phase measurements and subtracting the second phase measurement from the first, we have

$$(2\rho_V + \phi_{22} + \xi) - (\rho_V + \rho_H + \phi_{12} + \xi) = (\phi_{22} - \phi_{12}) + (\rho_V - \rho_H)$$

$$= (\psi_S - \theta_S) + (\rho_V - \rho_H); \tag{28}$$

Similarly, subtracting the third from the first, we get

$$(2\rho_V + \phi_{22} + \xi) - (2\rho_H + \phi_{11} + \xi) = (\phi_{22} - \phi_{11}) + 2(\rho_V - \rho_H)$$

$$= \psi_S + 2(\rho_V - \rho_H) \tag{29}$$

It is seen that the matrix phase angles contain errors due to ground plane polarization dependence. Because ρ_H and ρ_V have opposite signs this error will require correction, especially in the estimation of the matrix phase angle ψ_S. The ($\rho_V - \rho_H$) error terms can be eliminated from the matrix phase angle determinations by calibrating absolute phase measurements with the reference sphere. In this case ($\rho_V - \rho_H$) = $\Delta/2$ where Δ is the difference between phase angle measurements one and three.

A compensation scheme providing the element magnitude a_{12} without data manipulation requires equalization of reference sphere cross sections σ_{HH} and σ_{VV} through control of transmitted or received power. Compensation of phase angle determinations can also be effected during measurements.

AMPLITUDE AND RELATIVE-PHASE MEASUREMENTS

The required compensation of scattering-matrix measurements is easily accomplished during the normal equipment calibration procedure which employs a sphere as a reference. Though the same considerations apply here as those discussed immediately above under Amplitude and Absolute-Phase Measurements, the specific compensation procedure is again outlined since the present matrix determination technique is recommended for the RAT SCAT range.

Interpreting analysis presented in Section 3 under Amplitude and Relative-Phase Measurements, in terms of Equation 26, the following uncompensated measurements result:

	Trans.	Rec.		Measured Parameters	
(1)	Vert.	Vert. Horiz.	simul.	$\frac{(a_{22_{Target}}\Gamma_V^2)}{(a_{22_{Sphere}}\Gamma_V^2)}$ $\frac{(a_{12_{Target}}\Gamma_V\Gamma_H)}{(a_{22_{Sphere}}\Gamma_V^2)}$	$(\psi_S-\theta_S)+(\rho_V-\rho_H)$
(2)	If $a_{12} \neq 0$ proceed with the following measurements:				
	Horiz.	Horiz. Vert.	simul.	$\frac{(a_{11_{Target}}\Gamma_H^2)}{(a_{11_{Sphere}}\Gamma_H^2)}$ $\frac{(a_{12_{Target}}\Gamma_H\Gamma_V)}{(a_{11_{Sphere}}\Gamma_H^2)}$	$\theta_S+(\rho_V-\rho_H)$
(3)	If $a_{12} = 0$, replace 2 by				
	$\pi/4$	Horiz. Vert.	simul.	$\frac{(a_{11_{Target}}\Gamma_H^2)}{(a_{11_{Sphere}}\Gamma_H^2)}$ $\frac{(a_{22_{Target}}\Gamma_V^2)}{(a_{22_{Sphere}}\Gamma_V^2)}$	$\psi_S+2(\rho_V-\rho_H)$

A compensation procedure employing reference sphere calibrations is now outlined. In the relative phase measurement obtained in step 3, set the measured value to zero. Since ψ_S for the sphere is zero, this removes the ($\rho_V - \rho_H$) error term from all phase angle determinations. In the magnitude measurement obtained in step 3, set the receiver gain such that equal amplitudes are measured on both channels. This eliminates the dependence of amplitude measurements upon the parameters Γ and Γ_V . Measurements leading to the determination of scattering matrices of arbitrary targets no longer relate to the effective matrix S', but rather to the true matrix S. Because the present matrix determination technique involves a dual channel receiving capability, all matrix parameters are derived in a relative rather than absolute sense. Thus the above compensation procedure applies most effectively when amplitude and relative phase measurements are specified.

AMPLITUDE MEASUREMENT

Two formal compensation techniques exist. The antenna polarization compensation method, mentioned earlier and applicable to any measurement correction, may be employed here. The second technique involves measurement of the apparent target matrix S' and the ground plane transformation T to allow derivation of S on a computer.

SECTION 6

SCATTERING-MATRIX MEASUREMENTS ON THE RAT SCAT RANGE

UTILITY OF SCATTERING MATRIX MEASUREMENTS ON RAT SCAT

Once the question of the desirability of scattering-matrix data has been resolved, the utility of performing scattering-matrix measurements on the RAT SCAT range relates directly to the ease and accuracy with which scattering-matrix parameters are obtained. The complexity of a scattering matrix solution has been treated in Section 3. It was shown that matrix-determination techniques specifying phase angle measurements reduce to acceptable complexity in terms of range operation.

The utility of performing scattering-matrix measurements on the RAT SCAT range may be limited if this measurement facility is tied to a specific task, i.e., acceptance measurements. In this case targets to be measured before deployment would be studied at but one or two specific polarization combinations. However, if one considers the fact that future radars will employ antenna polarizations as yet undetermined, scattering matrix measurements provide the best means of insuring applicable cross-section data.

Discrimination experts hope to accomplish their task through use of many unrelated signatures, a group of which are classed as radar signatures. One promising signature from this group is based upon the polarization-transforming properties of potential targets. Several forms of partial polarization signatures now exist. Knowledge of the target-scattering matrix would facilitate comparison of these partial signatures and could be applied in the generation of new polarization signatures.

Determination of the target-scattering matrix may be viewed as a systematic approach to range measurement programs. Measurement procedures which lead to the scattering matrix solution provide more information than an equivalent number of non-coherent range measurements. (Knowledge of the scattering matrix allows calculation of target cross section for any combination of transmitting and receiving antenna polarizations. Expressions for some of the commonly desired cross sections are derived in Appendix A.) Repeated requests for cross-section data on a particular target at new polarization combinations could justify initial measurement of the scattering matrix. Considering the varied tasks in obtaining cross-section results, i.e., target transportation, target positioning and control, it appears that one additional request warrants scattering matrix measurements at the start.

TECHNIQUE RECOMMENDED FOR SCATTERING MATRIX MEASUREMENTS ON RAT SCAT

In the solution of target-scattering matrices, the measurement program reduces to moderate length when equipment capability allows phase measurement. Matrix determinations involving phase measurements are easily made independent of the field perturbation associated with ground-plane range operation. Planned RAT SCAT capability includes measurement of both absolute and relative phase. Thus the third method of matrix determinations involving amplitude measurements only is not considered further: a choice between methods requiring absolute and relative phase measurements will be made based upon the ease and accuracy with which the matrix solution is obtained.

The method of measuring amplitude and absolute phase is chosen if solution ease is emphasized: an additional cross-section measurement is involved with the relative phase approach. However, the dual-channel equipment required for relative phase measurements offers greater accuracy through balancing techniques. For this reason the second method of determining the target scattering matrix is specified; amplitude and relative phase measurements will maximize range accuracy with little loss of operational ease. The above choice is subject to attainment of sufficient isolation between receiving channels.

DATA HANDLING CONSIDERATIONS

Cross-section measurements are commonly performed over a complete azimuth rotation of the target. It is anticipated that measurements leading to the scattering matrix solution will be obtained in a similar manner. Angular equivalence between several measurements is necessary in order to extract coherent matrix information. Digital readout of measurement data will provide better aspect-angle correspondence than analog plots. Digital techniques also provide more accurate means of data reduction. It is recommended that both digital and analog forms of data readout be incorporated. Analog plots will indicate target aspect angles at which scattering matrix determinations will be meaningful. The foregoing statement recognizes the fact that inferior matrix information can result when element magnitudes are read from aspect nulls of the measured cross sections.

SECTION 7

CONCLUSIONS AND RECOMMENDATIONS

The following conclusions are enumerated:

1. The scattering matrix can be determined by at least three distinct experimental techniques. The first is the most direct method, involving the transmission of two linear polarizations and the reception of three linear polarizations. Measurement of absolute phase as well as amplitude of the three received signals is required.

   The second technique requires transmission of two linear polarizations and reception of four linear polarizations. However, only the determination of relative phase between the four received signals is necessary, as well as the amplitude of each.

   The third technique requires the transmission of two linear polarizations and the reception of seven linear polarizations. Measurements are restricted to the amplitude of each received signal.

2. The precision of computing any cross section from the scattering matrix depends primarily upon the precision with which the phase angles of matrix coefficients have been determined. Small errors in phase angles, arising from experimental error in matrix determination, can lead to gross error in computed cross section.

3. The ground plane introduces amplitude measurement errors for polarizations other than the two principal polarizations, horizontal and vertical. Other polarizations require correction which, for direct measurement of radar cross section, can be easily implemented.

4. The ground plane introduces errors which need correction if the scattering matrix is to be accurately determined. For measurement systems utilizing phase as well as amplitude in the matrix determination, compensation can be accomplished experimentally by a straightforward extension of the normal calibration procedure using a sphere reference. For the amplitude only technique, either complete knowledge of the ground plane transformation matrix is required, or a series of non-spherical reference

targets must be used during calibration to correct antenna polarizations for ground plane effects.

In view of the foregoing conclusions and knowledge of equipment complexity, it is recommended that instrumentation for matrix determination be confined to a relative phase and amplitude system.

REFERENCES

1. Sinclair, George, Modification of the Radar Range Equation for Arbitrary Targets and Arbitrary Polarization, Ohio State Univ. Res. Found., Report No. 302-19 (September 1948).

2. Tsu, Raphael, The Theory and Application of the Scattering Matrix for Electromagnetic Waves, Ohio State Univ. Antenna Laboratory Report No. 1073-2 (August 1960).

3. Graves, C. D., Radar Polarization Power-Scattering Matrix, University of Michigan Research Institute Report No. 2144-45-T (August 1955).

4. Kennaugh, Edward, Polarization Properties of Radar Reflections, Ohio State Univ. Res. Found., Report No. 389-12 (March 1952).

5. Bechtel, M. E., Project Acre Final Report, Cornell Aeronautical Laboratory Report No. UB-1405-P-1 (May 1960).

6. Crispin, J. W., et al, The Measurement and Use of Scattering Matrices, University of Michigan Rad. Lab., Report No. 2500-3-T (February 1961).

7. Huynen, J. R., Radar Target Sorting Based Upon Polarization Signature Analysis, (U) Lockheed Missiles and Space Division, Report No. LMSD-288216 (SECRET) (May 1960).

8. Deschamps, G. A., Geometrical Representation of the Polarization of a Plane Electromagnetic Wave, Proc. IRE, pp. 540-544 (May 1951).

9. Melling, W. P., An Analysis of Radar Cross Section Measurement Techniques, Cornell Aeronautical Laboratory Report No. UB-1088-P-104 (September 18, 1959).

10. Burrows and Attwood, Radio Wave Propogation, Academic Press (1949).

11. Kerr, Propagation of Short Radio Waves, Radiation Laboratory Series No. 13, McGraw Hill (1951)

APPENDIX A

CALCULATION OF CROSS-SECTION FROM SCATTERING MATRIX DATA

Once the scattering matrix of a target has been determined at a particular radar frequency, transmitter-receiver geometry, and target aspect angle, cross sections associated with any combination of transmitter and receiver antenna polarization may be calculated. This Appendix lists equations of commonly desired cross sections, notes relations between these cross sections, and indicates certain consistency procedures to be used in estimating range accuracy.

Employing Equations 3, 5, 9 and 11 of Sections 2 and 3,

$$\sigma_{pq} = \left| \hat{p}\, S\, \hat{q} \right|^2 \tag{A-1}$$

$$= \left| \begin{bmatrix} \cos\gamma_r & \sin\gamma_r e^{j\delta_r} \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} e^{j\theta_s} \\ a_{12} e^{j\theta_s} & a_{22} e^{j\varphi_s} \end{bmatrix} \begin{bmatrix} \cos\gamma_t \\ \sin\gamma_t e^{j\delta_t} \end{bmatrix} \right|^2 \tag{A-2}$$

$$\begin{aligned} &= a_{11}^2 \cos^2\gamma_t \cos^2\gamma_r + a_{22}^2 \sin^2\gamma_t \sin\gamma_r \\ &+ a_{12}^2 \left[\sin^2\gamma_t \cos^2\gamma_r + \cos^2\gamma_t \sin^2\gamma_r + \tfrac{1}{2} \sin 2\gamma_t \sin 2\gamma_r \cos(\delta_t - \delta_r) \right] \\ &+ \frac{a_{11} a_{22}}{2} \sin 2\gamma_t \sin 2\gamma_r \cos(\varphi_s + \delta_t + \delta_r) \\ &+ a_{11} a_{12} \left[\sin 2\gamma_t \cos^2\gamma_r \cos(\theta_s + \delta_t) + \cos^2\gamma_t \sin 2\gamma_r \cos(\theta_s + \delta_r) \right] \\ &+ a_{22} a_{12} \left[\sin^2\gamma_t \sin 2\gamma_r \cos(\theta_s - \varphi_s - \delta_r) + \sin 2\gamma_t \sin^2\gamma_r \cos(\theta_s - \varphi_s - \delta_t) \right] \end{aligned} \tag{A-3}$$

Consider linear polarization combinations for the present. Then $\delta_t = \delta_r = 0$. Two polarization combinations of interest result from parallel linear antennas and crossed linear antennas. For the parallel case $\gamma_t = \gamma_r$ and the cross section σ_{pp} obtained from Equation A-3 is

$$\begin{aligned} \sigma_{pp}(\gamma) &= a_{11}^2 \cos^4\gamma + a_{22}^2 \sin^4\gamma + \left[4a_{12}^2 + 2a_{11} a_{22} \cos\varphi_s \right] \cos^2\gamma \sin^2\gamma \\ &+ 4a_{11} a_{12} \cos\theta_s \cos^3\gamma \sin\gamma + 4a_{22} a_{12} \cos(\theta_s - \varphi_s) \sin^3\gamma \cos\gamma \end{aligned} \tag{A-4}$$

For the crossed case, $\gamma_r = \gamma_t \pm \pi/2$, we have

$$\sigma_{CP}(\gamma) = \left[a_{11}^2 + a_{22}^2 - 2a_{12}^2 - 2a_{11}a_{22}\cos\varphi_s\right]\sin^2\gamma\cos^2\gamma$$
$$+ a_{12}^2\left[\cos^4\gamma + \sin^4\gamma\right]$$
$$- \left[a_{11}a_{12}\cos\theta_s - a_{22}a_{12}\cos(\theta_s - \varphi_s)\right]\cos 2\gamma\sin 2\gamma \tag{A-5}$$

The angle γ above refers to the orientation of the transmitting antenna with respect to the horizontal. Circular polarizations obtain when $\gamma = \pi/4$ and $\delta = \pm\pi/2$. For $\delta = -\pi/2$ the antenna polarization is termed right circular; $\delta = \pi/2$ produces left circular polarization. Three cases arise: σ_{RR} (right circular transmission and reception), σ_{LL} (left circular transmission and reception), and σ_{LR} (right circular transmission with left circular reception). For σ_{LR} substitute $\gamma_t = \gamma_r = \pi/4$, $\delta_t = -\pi/2$, $\delta_r = \pi/2$ in A-3.

$$\sigma_{LR} = \sigma_{RL} = \frac{1}{4}\left[a_{11}^2 + a_{22}^2 + 2a_{11}a_{22}\cos\varphi_s\right] \tag{A-6}$$

To obtain σ_{RR} put $\gamma_t = \gamma_r = \pi/4$, $\delta_t = \delta_r = -\pi/2$

$$\sigma_{RR} = \frac{1}{4}\left[a_{11}^2 + a_{22}^2 + 4a_{12}^2 - 2a_{11}a_{22}\cos\varphi_s + 4a_{11}a_{12}\sin\theta_s - 4a_{22}a_{12}\sin(\theta_s - \varphi_s)\right] \tag{A-7}$$

Similarly for σ_{LL} substitute $\delta_t = \delta_r = +\pi/2$

$$\sigma_{LL} = \frac{1}{4}\left[a_{11}^2 + a_{22}^2 + 4a_{12}\ \ 2a_{11}a_{22}\cos\varphi_s - 4a_{11}a_{12}\sin\theta_s + 4a_{22}a_{12}\sin(\theta_s - \varphi_s)\right] \tag{A-8}$$

An interesting relation between cross sections results from the fact that the trace of a symmetrical matrix is a constant with respect to rotation.

$$a_{11}^2 + a_{22}^2 + 2a_{12}^2 \quad \text{constant}$$
$$\sigma_{PP}(\gamma) + \sigma_{PP}(\gamma \pm \tfrac{\pi}{2}) + 2\sigma_{CP}(\gamma)$$
$$= \sigma_{RR} + \sigma_{LL} + 2\sigma_{RL} \tag{A-9}$$

Under the constraint that the target has a horizontal plane of symmetry containing the radar line of sight, the scattering matrix is diagonalized: $a_{12} = 0$. Referring to such a body as a symmetrical target,

$$S_{SYM.} = \begin{bmatrix} a_{11} & 0 \\ 0 & a_{22} e^{j\varphi_s} \end{bmatrix} \tag{A-10}$$

and the following equations hold:

$$\sigma_{PP}(\gamma)_{SYM} = a_{11}^2 \cos^4 \gamma + a_{22} \sin^4 \gamma + 2a_{11} a_{22} \cos\varphi_s \sin^2\gamma \cos^2\gamma \tag{A-11}$$

$$\sigma_{CP}(\gamma)_{SYM.} = \left[a_{11}^2 + a_{22}^2 - 2a_{11} a_{22} \cos\varphi_s\right] \sin^2\gamma \cos^2\gamma \tag{A-12}$$

$$\sigma_{RR\,SYM.} = \sigma_{LL\,SYM} = \tfrac{1}{4}\left[a_{11}^2 + a_{22}^2 - 2a_{11} a_{22} \cos\varphi_s\right] \tag{A-13}$$

$$\sigma_{RL\,SYM} = \sigma_{LR\,SYM} = \tfrac{1}{4}\left[a_{11}^2 + a_{22}^2 + 2a_{11} a_{22} \cos\varphi_s\right] \tag{A-14}$$

$$= \sigma_{RL} \tag{A-15}$$

From the above equations, it is seen that, for a symmetrical target,

$$\sigma_{PP\,SYM}\left(\pi/4\right) = \sigma_{RL\,SYM} = \sigma_{RL} \tag{A-16}$$

$$\sigma_{CP\,SYM}\left(\pi/4\right) = \sigma_{RR\,SYM} \tag{A-17}$$

The above relations afford a check on measured cross section. However, they do not reflect on the accuracy of a scattering-matrix determination. In Section 3, the techniques for the solution of the scattering matrix were treated in terms of the minimum number of measurements required. The first additional redundant measurement aims at improving the accuracy of the a_{12}, θ_s measurement. In general, this element is smallest and most requires correction. It is also the easiest element evaluation to upgrade. For the method involving measurement of amplitude and phase, an additional vertical-polarization reception of the horizontal transmission provides a separate estimate of a_{12} and θ_s. The agreement between corresponding terms reflects on range accuracy.

In order to establish the validity of range data, it is usual to perform prove-in measurements. The following procedure allows a check on the accuracy of a scattering-matrix determination. The scattering matrix would be measured in two reference systems; the original horizontal-vertical system and, for example, a new reference rotated $\pi/4$ degrees to the original basis. A simple rotational transformation of data measured in the new reference system gives a check on matrix measurement accuracy.

A P P E N D I X   B

S C A T T E R I N G   M A T R I X   E R R O R   A N A L Y S I S

The parameters comprising the elements of the scattering matrix must be experimentally determined, hence are subject to inaccuracy. Calculation of cross section from the scattering matrix propagates the element errors in a complicated manner. The relationship between errors in calculated cross sections (for various meaningful polarization combinations) and matrix-element inaccuracies was investigated on the 704 computer. Analysis described here assumes that the matrix was derived from a system utilizing direct phase measurement, since preliminary investigation has indicated that this should be the recommended technique. Because element inaccuracies are dependent upon the particular procedure followed for matrix determination, a similar analysis should be made on an amplitude-only measurement system, such as that outlined in Section 3, in order to make this recommendation firm.

In order that results be significant to RAT SCAT, the scattering matrices studies should have characteristics common to the class of potential RAT SCAT targets. Such a set of scattering-matrix data is contained in Reference 6. Matrix data of interest here result from measurements at 9.7 gigacycles on two particular bodies: a one-foot model of a Jupiter-type missile, and a one-foot Jupiter model with fins. Ten scattering-matrices were chosen for study, five matrix solutions applying to five distinct aspect angles for each target. (See pages 46 and 47 of Reference 6.) Alterations to the measured parameters have been effected in instances where the proper relationships between element magnitudes and element phase angles are known. For example, the parameter 12 should be zero for a target exhibiting a horizontal plane of symmetry. This value is substituted in Table B-1 for the missile without fins. Table B-1 ("True Matrix Parameters") contains the matrix parameters on which analysis is based; because representative matrix data suffices for the present purpose, these measured data are hereafter assumed exact.

The ten scattering matrices may logically be grouped into three classes. Class-A matrices correspond to specular reflection: $a_{11} = a_{22}, \psi_s = 0, a_{12} = 0$. Matrices numbered 1, 5, 6 and 10 belong in Class A. Class-B matrices exhibit zero off-diagonal elements: $a_{12} = 0$. Matrices 2, 3 and 4 are Class-B matrices. Class C refers to a general scattering matrix with none of the above restrictions. Matrices 7, 8 and 9 comprise Class C. The description of results is facilitated by reference to the above

TABLE 9-1

TRUE MATRIX PARAMETERS

MATRIX CLASS	MATRIX NO.	AZIMUTH ASPECT ANGLE - degrees	a_{11}^2 sq meters	a_{22}^2	a_{12}^2	θ_s degrees	φ_s (degrees)
MISSILE WITHOUT FINS							
A	1	0	1.79×10^{-3}	1.79×10^{-3}	0	0	0
B	2	40	4.50×10^{-4}	2.26×10^{-3}	0	0	-236
B	3	90	5.70	4.52	0	0	+16
B	4	110	9.00×10^{-3}	1.79×10^{-2}	0	0	+100
A	5	180	2.86×10^{-1}	2.86×10^{-1}	0	0	0
MISSILE WITH FINS							
A	6	0	1.79×10^{-2}	1.79×10^{-2}	0	0	0
C	7	30	9.00×10^{-3}	5.95×10^{-2}	7.1×10^{-4}	-89	± 61
C	8	90	9.00	9.00	4.5×10^{-2}	+53	± 4
C	9	120	1.43×10^{-1}	2.84×10^{-2}	7.14×10^{-3}	+60	-7
A	10	180	1.13×10^{-1}	13×10^{-1}	0	0	0

classification scheme.

The next requirement is a realistic relationship between exact matrix parameters and matrix parameters measured on a cross-section range. Errors in the measurement of element magnitudes are considered to arise from three sources: gain variations, background, and receiver noise. Gain variations are represented by the factor ΔG in the expression for gain, $G_0 + \Delta G$, where G_0 is the true gain. ΔG is assumed constant over a set of matrix measurements; that is, short-term gain variations are neglected with respect to the larger long-term variation. The background disturbance is commonly represented by an equivalent cross section σ_b. Since background interaction with the true scattered signal involves vector fields, a maximum error will occur when the signal and background amplitudes reinforce or oppose one another. The measured cross section, σ_m, may be written

$$\sigma_m = \left(\frac{G_0 + \Delta G}{G_0}\right)^2 \left(\sqrt{\sigma_t} \pm \sqrt{\sigma_b}\right)^2 \tag{B-1}$$

Noise in the receiver front end will be essentially random. Measurement error from this source properly would involve an RMS noise error predicated on the assumption that the phase difference between the noise and the received signal is completely random. This simple study assumes noise effects represented by σ_N in the following manner:

$$\sigma_m = \left(\frac{G_0 + \Delta G}{G_0}\right)^2 \left[\left(\sqrt{\sigma_t} \pm \sqrt{\sigma_b}\right)^2 + \sigma_N\right] \tag{B-2}$$

The minimum resolvable cross section obtains for

$$\sigma_{m_{min\ res}} = \left(\frac{G_0 + \Delta G}{G_0}\right)^2 \left[\sigma_b + \sigma_N\right] \tag{B-3}$$

Equations B-2 and B-3 were used to obtain Figure B-1 relating measurement error (σ_m / σ_t in db) and cross-section magnitude ($\sigma_t / \sigma_{m_{min\ res}}$ in db). Arbitrary values of σ_b and σ_N were chosen: $\sigma_b = \sigma_N = 10^{-4}$ square meter. ΔG was chosen to be zero. The effects of long-term gain variations may be included in the following analysis simply by adding the gain variation in db to the cross-section error predicted for $\Delta G = 0$. Figure B-1 corresponds to Figure 1 in Reference 9; a comparison of these curves indicates the effects of the noise term. Measured cross sections are within $\pm$ 1 db of the true cross section when the true cross section exceeds the minimum resolvable cross section by 16 db or more. All measured values of element magnitudes are subject to the errors described in Figure B-1.

Representative errors in phase measurements were chosen independently of the true value of the matrix phase angle. Measurement errors at microwave frequencies may well exceed one degree; consequently, nominal measurement errors of $\pm$ 2, $\pm$ 5 and $\pm$ 10 degrees were chosen for their investigation. These matrix phase-angle errors may be attributed to matrix-determination techniques involving measurements of either absolute or relative phase angles.

ANALYSIS

Three separate programs were run. In the first study, measured phase angles were set equal to true phase angles so that the errors in calculated cross section resulted from element magnitude errors only. The second program assumed correct element magnitudes and the effect of phase-angle errors was investigated. The final program allows magnitude and phase-angle errors in simulation of actual range performance.

Propagation of matrix-element errors was examined by comparing cross sections calculated from corresponding exact and (simulated) measured scattering matrices. The following meaningful cross sections, introduced in Appendix A, were calculated: $\sigma_{pp}(\gamma)$, $\sigma_{cp}(\gamma)$, σ_{RR} , σ_{LL} and σ_{RL} . The two linear-polarization cross sections retain γ dependence, where γ is the transmitting-antenna orientation with respect to the horizontal.

PROGRAM 1. ELEMENT MAGNITUDE ERRORS

Measured scattering matrices are generated from the true scattering matrices listed in Table B-1 by introducing worst-case measurement errors according to the curves of Figure B-1. Because measured elements may be greater or less than the true element, two distinct matrices are generated for each true matrix. Calculated cross sections relating to the measured matrix generated by the upper-bound curve of Figure B-1 are denoted by the symbol $\oplus$;

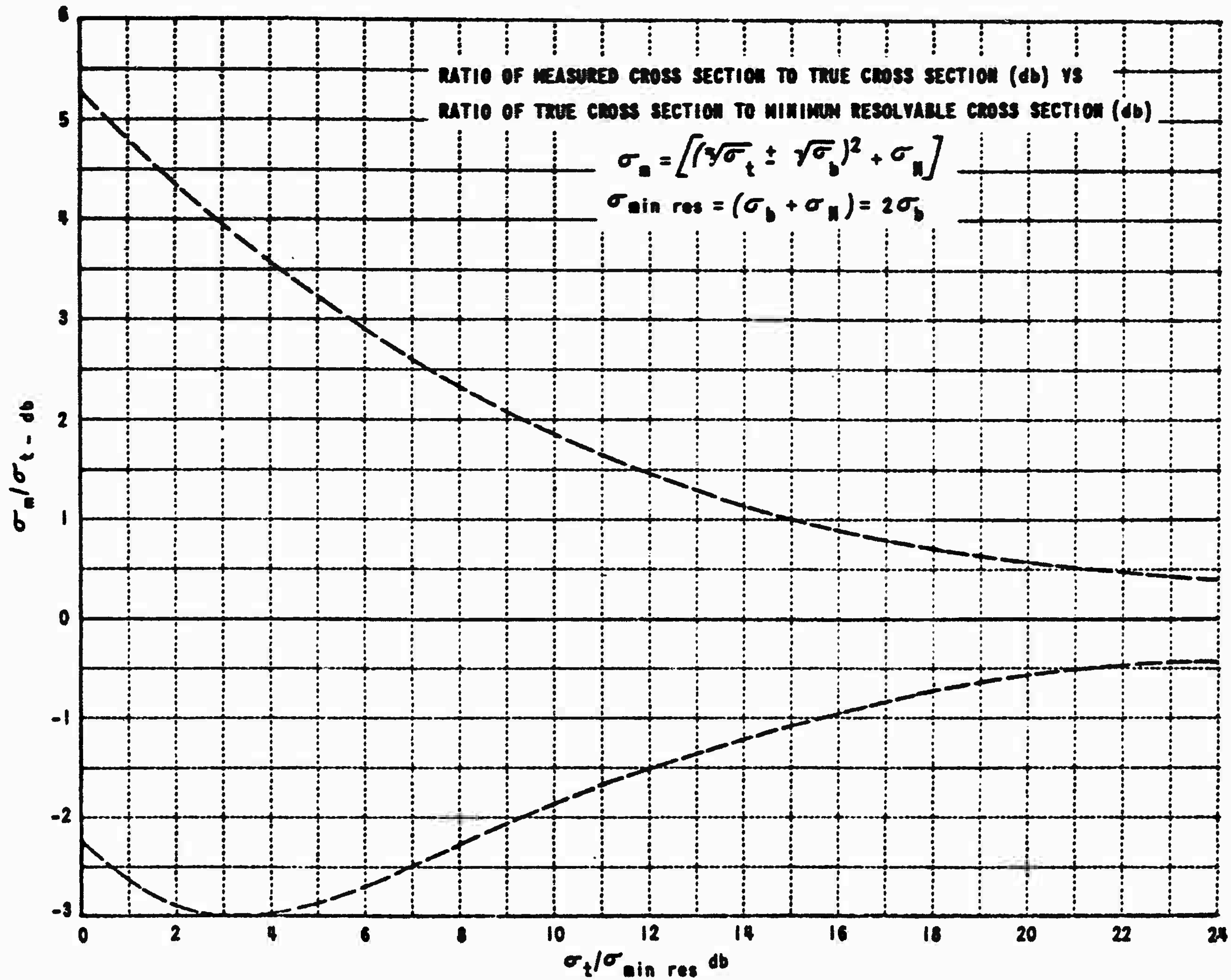


Figure B-1 MEASUREMENT ERROR BOUND

results associated with the lower bound are subscripted Θ . Matrix phase angles are assumed correct in this program.

Investigation of $\sigma_{pp_c}(\gamma)$

The equation for the calculated cross section $\sigma_{pp}(\gamma)$ is obtained from Equation A-4 by substituting a_{ij_m} , the measured element magnitude, for a_{ij} m the exact value.

$$\sigma_{pp_c}(\gamma) = a_{11_m}^2 \cos^4\gamma + a_{22_m}^2 \sin^4\gamma + \left[4a_{12_m}^2 + 2a_{11_m} a_{22_m} \cos\varphi_s\right] \sin^2\gamma \cos^2\gamma$$
$$+ 4a_{11_m} a_{12_m} \cos\theta_s \cos^3\gamma \sin\gamma + 4a_{22_m} a_{12_m} \cos(\theta_s - \varphi_s) \sin^3\gamma \cos\gamma$$

(B-4)

Class-A matrices produce $\sigma_{pp_t}(\gamma) = a_{11}^2$, a constant. The calculated cross section is

(B-5)

$$\sigma_{pp_c}(\gamma)_{CLASS\ A} = a_{11_m}^2 + 4a_{12_m}^2 \sin^2\gamma \cos^2\gamma + 4a_{11_m} a_{12_m} \sin\gamma \cos\gamma$$

where

$$a_{12}^2 = \sigma_{m_{min.\ res}} = 2\sigma_b \quad \text{since } a_{12} = 0. \text{ For } \gamma = 0 \text{ or } \pi/2,$$

the calculated error bound follows the measurement error curve since only one element comes into play. The calculated error is maximum when $\gamma = \pi/4$ or $3\pi/4$ depending upon whether measurements of elements are greater or less than the true element magnitude. Results are shown in Figure B-2. Computed results for matrices 1, 5, 6 and 10 are shown.

Class-B matrices produce errors in $\sigma_{pp_c}(\gamma)$ according to Figure B-3. That the maximum error no longer occurs at $\gamma = \pi/4$ or $3\pi/4$ is shown by tracing the γ dependence of the plots. The characteristic clover-leaf pattern is due to the fact that the cross section $\sigma_t(\gamma)$, is symmetrical about $\gamma = \pi/2$. The patterns cross the measurement-error-bound curve at $\gamma = 0, \pi/2$ and π because, again, only one element comes into the calculated cross section $\sigma_{pp_c}(\gamma)$ at these γ's . Plots are shown for matrices 2 and 4. Matrix 3 produces abscissa values on the order of 40 db. The calculated errors lie inside the outer bounds depicted in

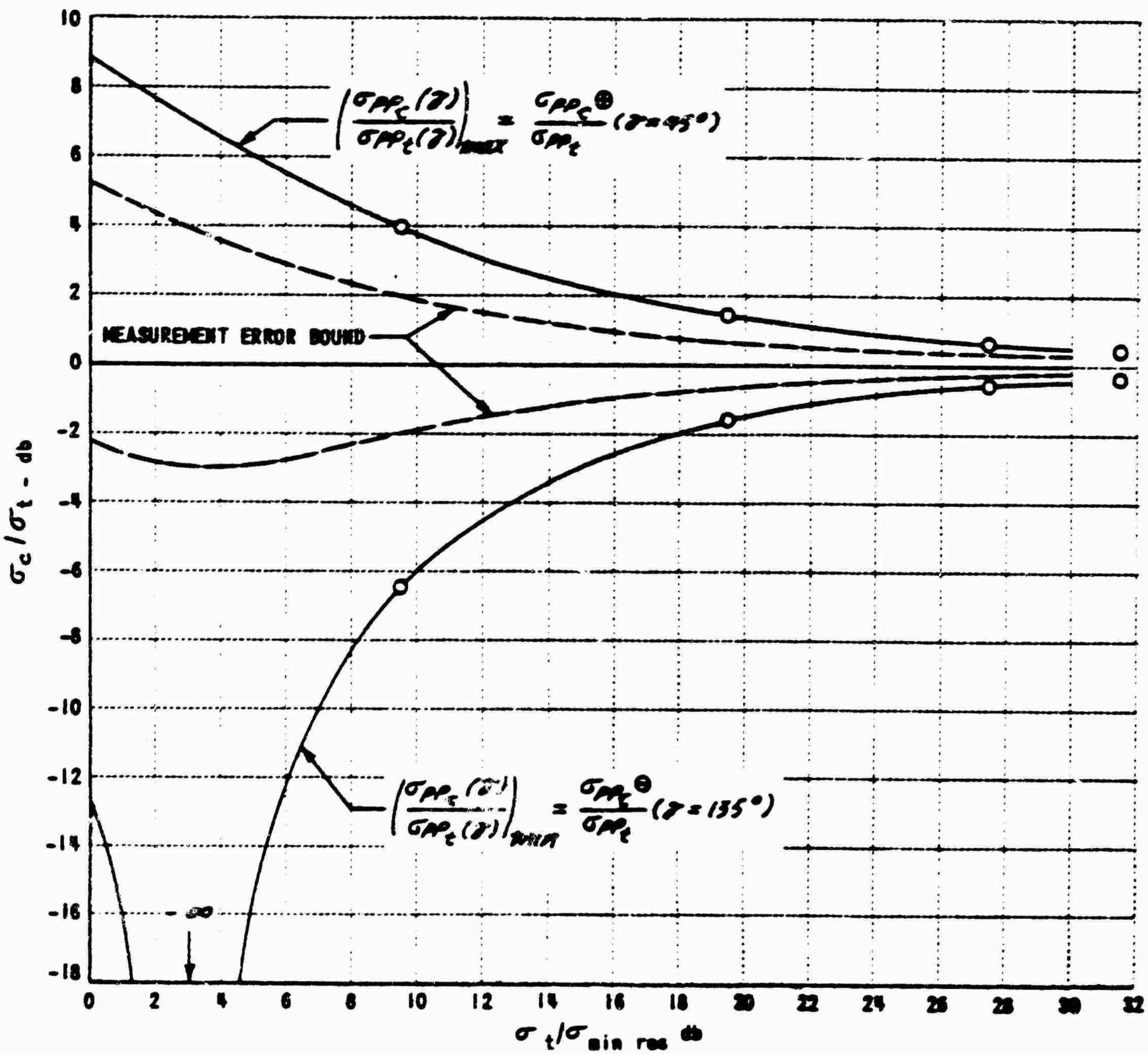


Figure B-2 PROGRAM I - ELEMENT MAGNITUDE ERRORS, σ_{pp_c} INVESTIGATION, CLASS A MATRICES

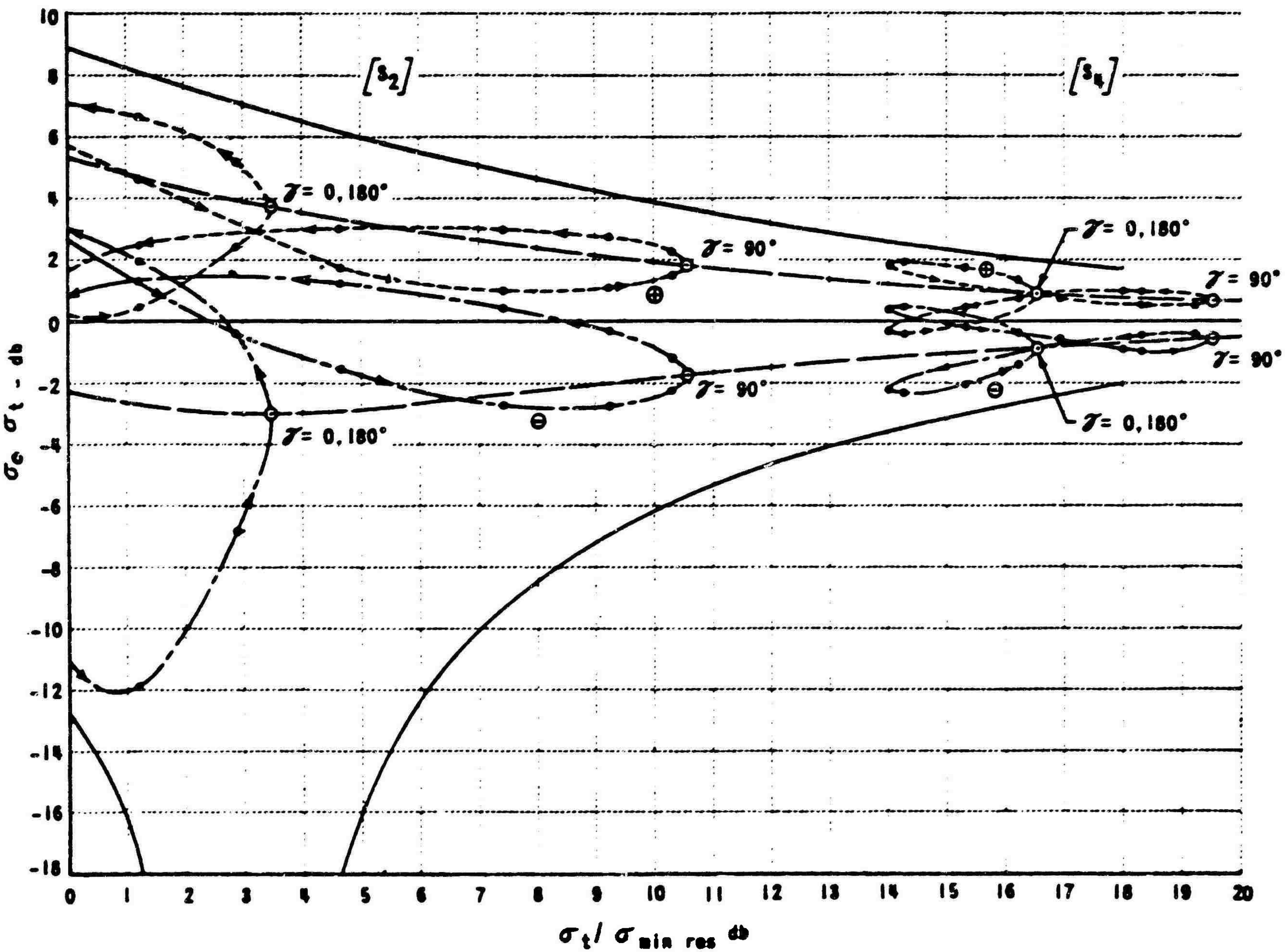


Figure B-3 PROGRAM I - ELEMENT MAGNITUDE ERRORS, $\sigma_{pp_c}(\gamma)$ INVESTIGATION, CLASS B MATRICES

Figure B-2; however, it is inconvenient to plot the small values.

Class-C matrices give rise to the simple loop plots shown in Figure B-4. The general matrix does not produce $\sigma_{pp_t}(\gamma)$ symmetry in the region $0 \leq \gamma \leq 180^{o}$. Again, matrix 8 results lie far to the right on the abscissa; it is sufficient to note that the plots lie inside the outer bounds generated by Class-A matrices.

It is believed that the outer bounds generated by the Class-A analysis represent a maximum bound on calculated errors under the restriction of Program 1. All matrices investigated suggest this to be the case. Search for analytical evidence to support this belief is beyond the scope of the present contract.

Investigation of $\sigma_{cp_c}(\gamma)$

The analysis concerns Equation A-5 written in the following form:

$$\begin{aligned}\sigma_{cp_c}(\gamma) = &\left[a_{11_m}^2 + a_{22_m}^2 - 2a_{12_m}^2 - 2a_{11_m} a_{22_m} \cos \psi_s\right] \sin^2\gamma \cos^2\gamma \\ &+ \left[\cos^4\gamma + \sin^4\gamma\right] a_{12_m}^2 \\ &- \left[a_{11_m} a_{12_m} \cos\theta_s - a_{22_m} a_{12_m} \cos(\theta_s - \psi_s)\right] \cos 2\gamma \sin 2\gamma\end{aligned} \qquad \text{(B-6)}$$

Class-A matrices result in zero true cross section independent of the orientation angle γ. The calculated cross section, obtained for $a_{11_m} = a_{22_m}, a_{12_m} =$ minimum resolvable signal, $\theta_s = \psi_s = 0$, becomes:

$$\sigma_{cp_c}(\gamma) = a_{12_m}^2 \cos^2 2\gamma = \sigma_{min\,res.} \cos^2 2\gamma \qquad \text{(B-7)}$$

Since the true cross section is zero, the maximum error in calculated cross section is $\sigma_{min\,res}$. This maximum error occurs for $\gamma = 0$ and $\pi/2$. Class-B matrices produce normalized errors shown in Figure B-5. All error curves lie within the calculated error bound generated by $\sigma_{pp_c}(\pi/4)$ for Class-A matrices. Figure B-6 contains Class-C error curves for $\sigma_{cp_c}(\gamma)$. Again error curves are well within the outer error bound on calculated cross sections.

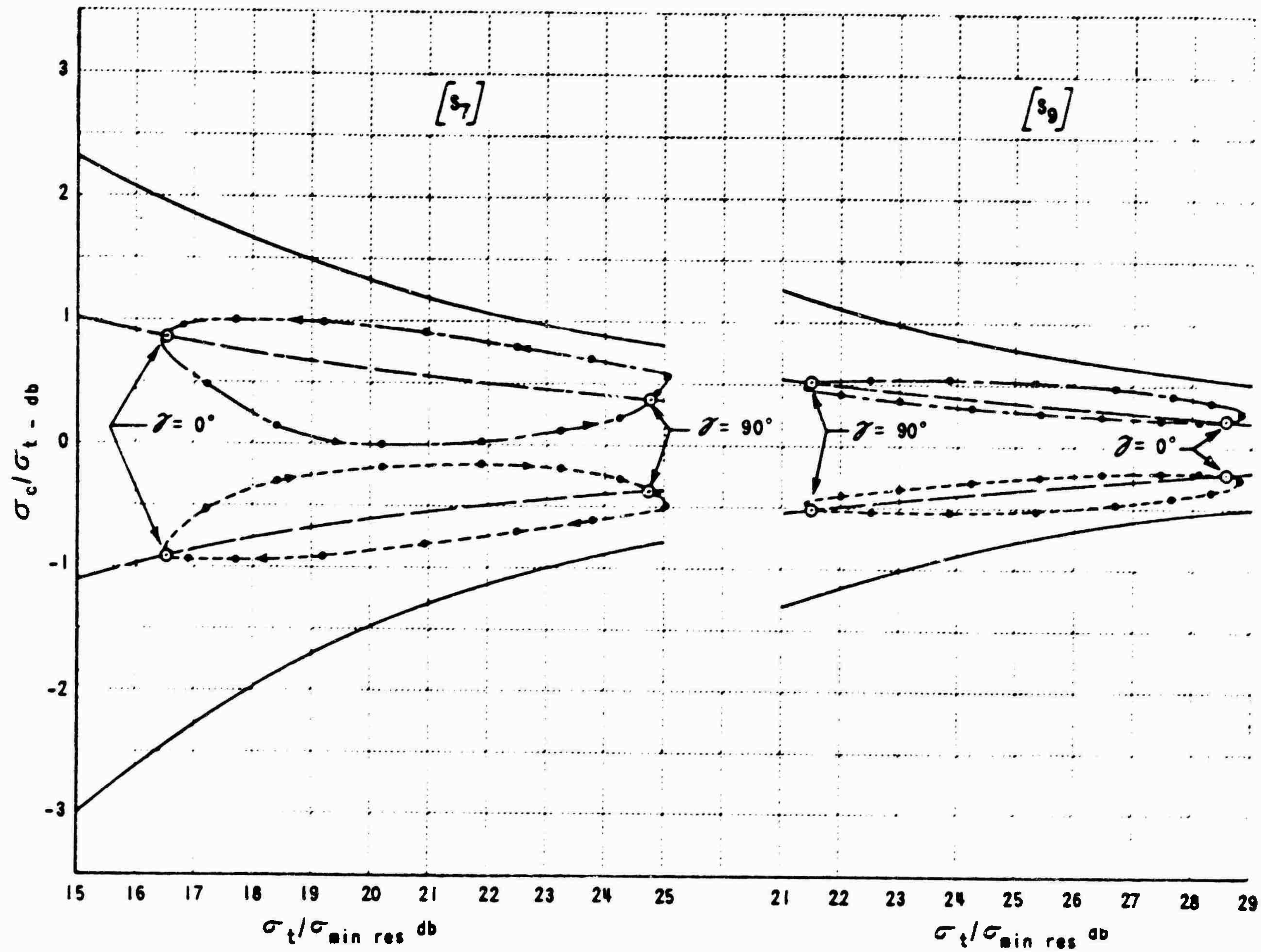


Figure B-4 PROGRAM I - ELEMENT MAGNITUDE ERRORS, $\sigma_{FP_C}(\gamma)$ INVESTIGATION, CLASS C

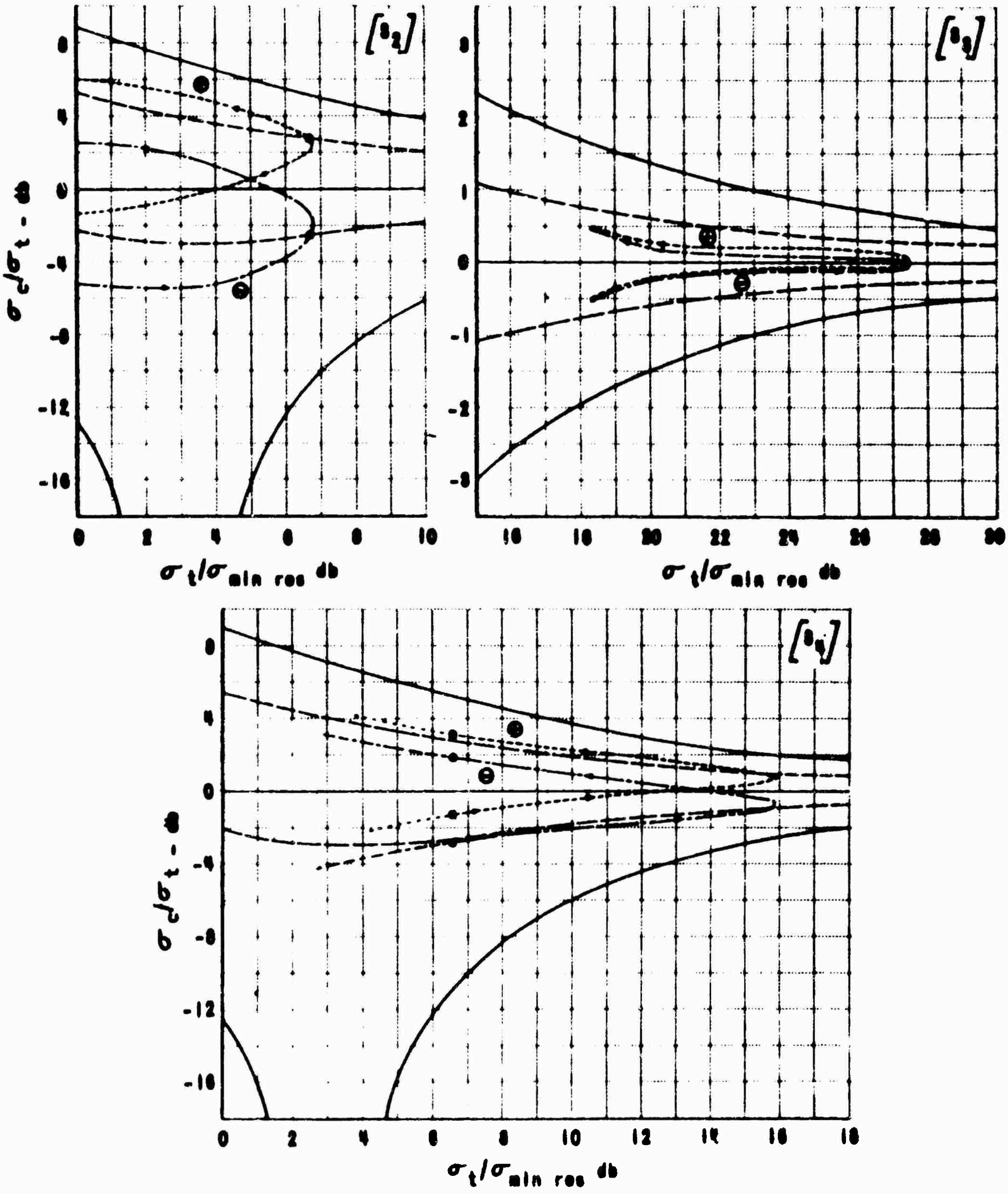


Figure B-5 PROGRAM I - ELEMENT MAGNITUDE ERRORS, $\sigma_{cp_c}(\gamma)$ INVESTIGATION, CLASS B

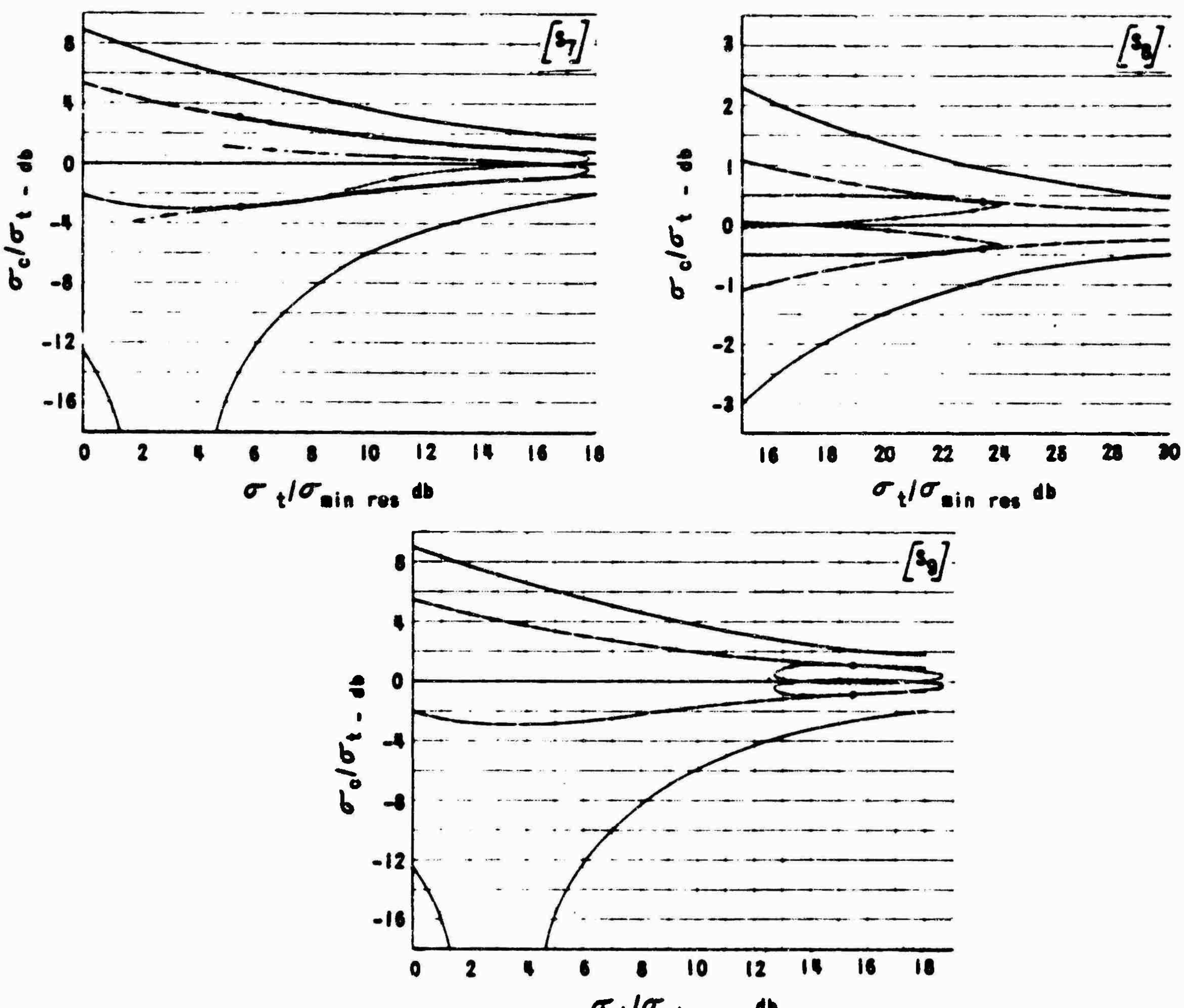


Figure B-6 PROGRAM I - ELEMENT MAGNITUDE ERRORS, $\sigma_{\varphi_c}(\vartheta)$ INVESTIGATION, CLASS C

Investigation of σ_{RL_c}

The equation of interest derives from Equation A-6.

$$\sigma_{RL_c} = 1/4\left[a_{11_m}^2 + a_{22_m}^2 + 2a_{11_m} a_{22_m} \cos \varphi_s\right] \tag{B-8}$$

Class-A matrices give $\sigma_{RL_c} = a_{11_m}^2$ so that matrices 1, 5, 6 and 10 produce errors lying on the original measurement-error-bound curves. Class-B matrices exhibit σ_{RL_c} errors which must lie within the above mentioned bound. The same is true of Class-C results.

Figure B-7 contains the original measurement error bounds along with computed values. Results of matrices 3, 5 and 8 lie within the bound but fall outside the abscissa range and are not plotted. σ_{RL_c} errors obey the measurement bound because the element a_{12_m}, whether a_{12} exists or results from a minimum resolvable cross section, does not enter into Equation B-8.

Investigation of σ_{RR_c}, σ_{LL_c}

Cross sections calculated for same-sense circular polarizations are treated together because of their similarity. Equations A-7 and A-8 become

$$\sigma_{c\,RR}^{LL} = \frac{1}{4}\Big[a_{11_m}^2 + a_{22_m}^2 + 4a_{12_m}^2 - 2a_{11_m} a_{22_m} \cos\varphi_s \mp 4a_{11_m} a_{12_m} \sin\theta_s$$
$$\pm 4a_{22_m} a_{12_m} \sin(\theta_s - \varphi_s)\Big] \tag{B-9}$$

where the sign of the last two terms is determined by the antenna polarization. For Class-A matrices, $\sigma_{RR_t} = \sigma_{LL_t} = 0$, and $\sigma_{RR_c} = \sigma_{LL_c} = \sigma_{min\ res}$. Class-B matrices result in error points limited only by the outer error bound. Results are shown in Figure B-8. Class-C matrices produce smaller errors as shown in the same figure; the bound on these error points is the original measurement error bound.

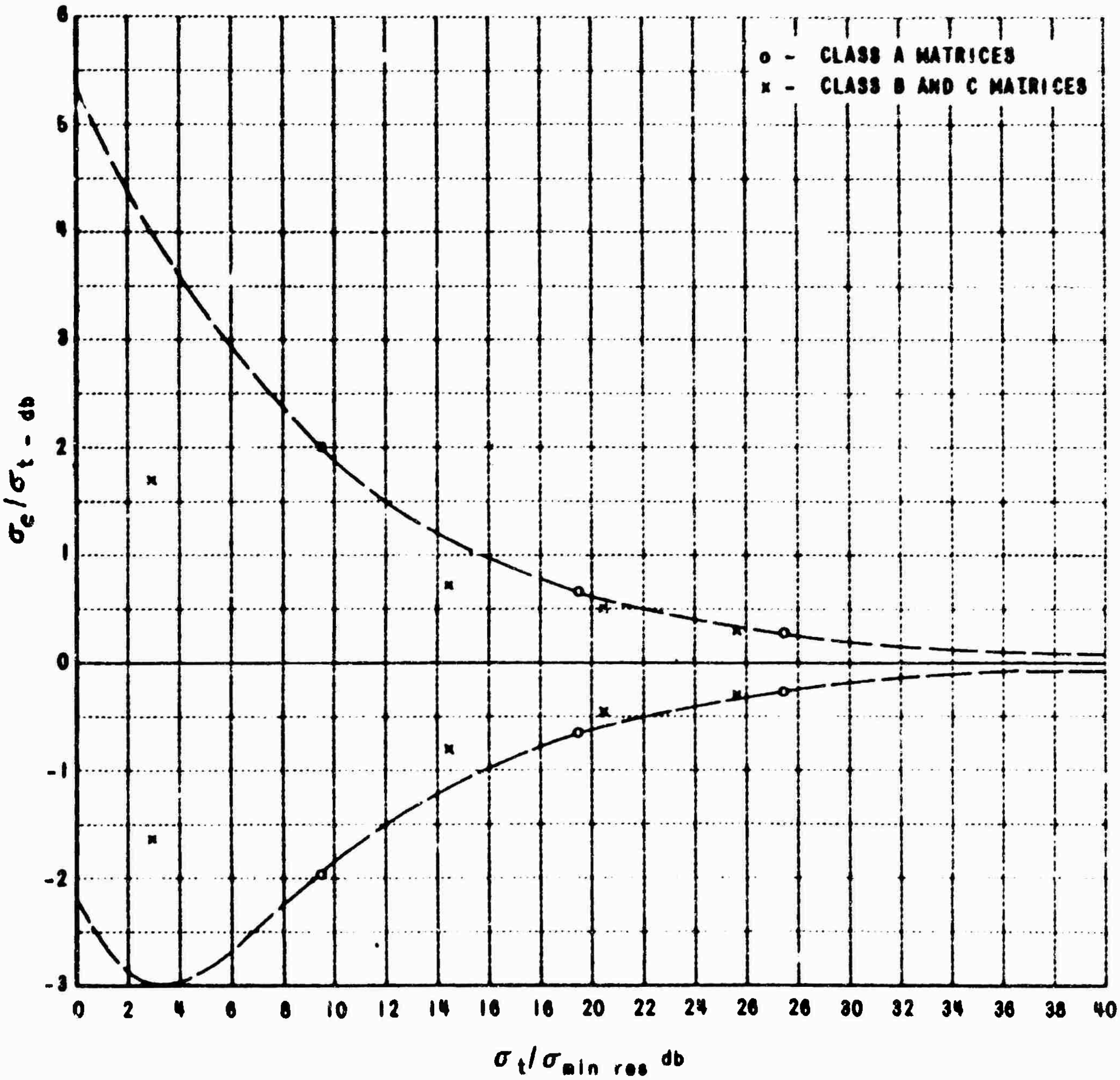


Figure B-7 PROGRAM I - ELEMENT MAGNITUDE ERRORS, σ_{RL_C} INVESTIGATION

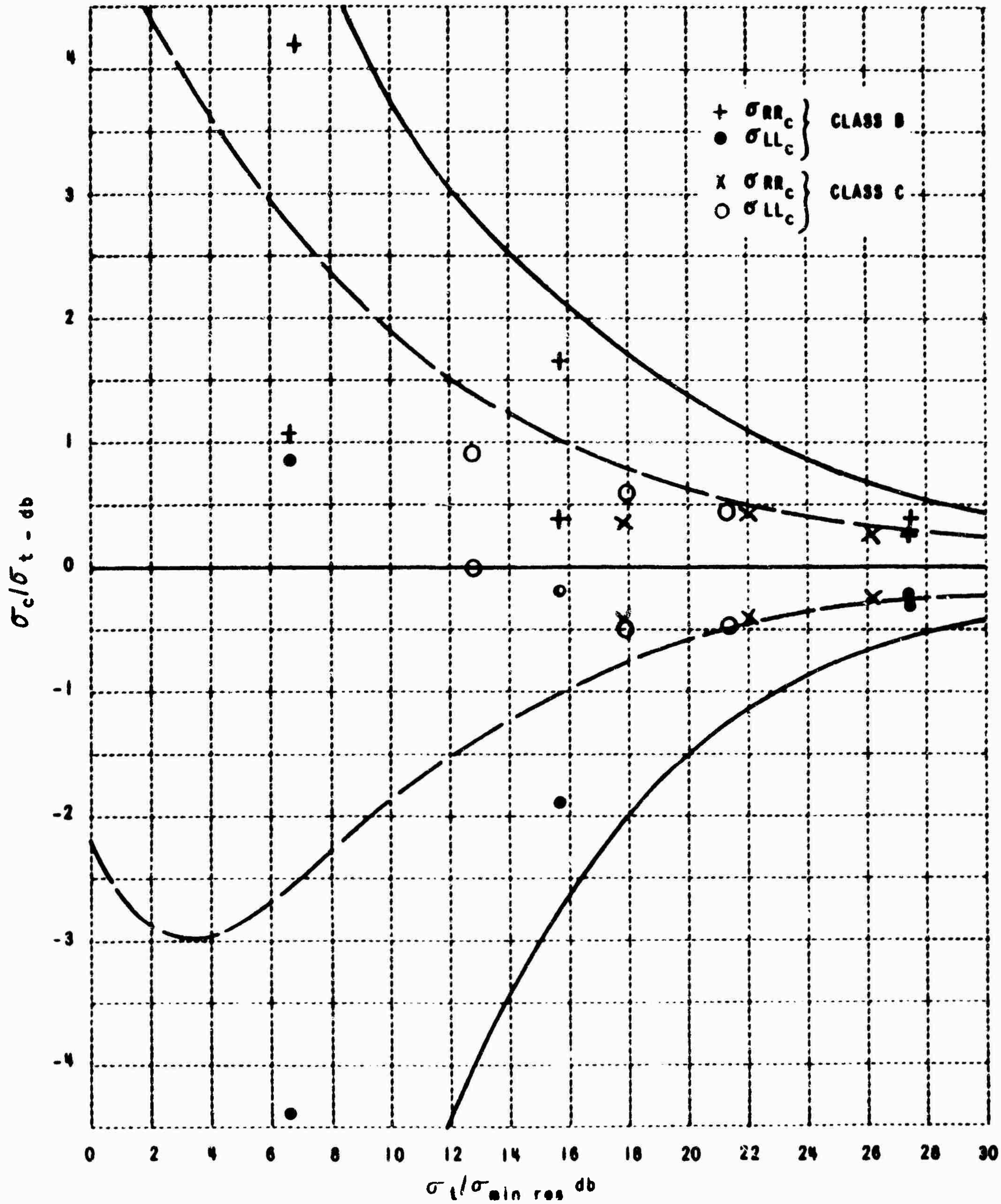


Figure B-8 PROGRAM I - ELEMENT MAGNITUDE ERRORS σ_{RR_c}, σ_{LL_c} INVESTIGATION

PROGRAM 2. ELEMENT PHASE ANGLE ERRORS

Measured scattering matrices were generated from the true matrices listed in Table B-1 by introducing phase angle errors of ± 2, ± 5 and ± 10 degrees in the matrix parameters θ_S and ψ_S. The magnitudes of the measured matrix elements are assumed correct during this program.

Data normalized relative to a minimum resolvable cross section has no meaning in the study of angular errors. However, this form of data presentation is retained here for convenience. Error bounds associated with the previous program are included to enable quick comparison of the effects of angular measurement errors with magnitude measurement errors previously discussed.

Investigation of $\sigma_{pp_c}(\gamma)$

Class-A matrices produce the following relation between calculated cross section and the angle error ξ associated with the ψ_s measurement:

$$\sigma_{pp_c} = \frac{a_{11}^2}{2}\left[1 + \cos\xi\right] \qquad \text{(B-10)}$$

The true cross section, a_{11}^2, is also indpendent of antenna orientation γ. Phase-angle errors of ± 30 degrees produce a -0.3 db error in the normalized cross section ratio $\sigma_{pp_c}/\sigma_{pp_t}$. It is seen that phase-angle errors will have negligible effect on σ_{pp_c} for Class-A matrices.

Class-B matrices involve calculation errors governed by the relation:

$$\frac{\sigma_{pp_c}(\gamma)}{\sigma_{pp_t}(\gamma)} = \frac{a_{11}^2\cos^4\gamma + a_{22}^2\sin^4\gamma + 2a_{11}a_{22}\cos^2\gamma\sin^2\gamma\cos(\psi_s + \xi)}{a_{11}^2\cos^4\gamma + a_{22}^2\sin^4\gamma + 2a_{11}a_{22}\cos^2\gamma\sin^2\gamma} \qquad \text{(B-11)}$$

The angular error associated with matrix parameter ψ_s is the only contribution to cross-section inaccuracy. The calculated cross section equals the true cross section for $\gamma = 0$ and $\pi/2$. The error in calculated cross section is maximum for $\gamma = \pi/4$.

$$\left(\frac{\sigma_{pp_c}}{\sigma_{pp_t}}\right)_{max} = \frac{a_{11}^2 + a_{22}^2 + 2a_{11}\, a_{22}\cos(\psi_s + \xi)}{a_{11}^2 + a_{22}^2 + 2a_{11}\, a_{22}\cos(\psi_s)} \qquad \text{(B-12)}$$

Results are shown in Figure B-9 for $\xi = \pm 10$ degrees. The smaller values of ξ produce error curves lying within the curves shown for $\xi = \pm 10$ degrees.

Class-C matrices introduce the effects of errors in both θ_s and ψ_s. Errors curves shown in Figure B-10 apply for $\xi = \pm 10$ degrees.

Matrices 3 and 8 produce error curves which also conform to the original measurement error bound but lie outside the plotting region. It is concluded that ± 10 degree errors in matrix phase angles result in calculated cross sections having errors comparable with those generated in the corresponding investigation of element magnitude errors. Lesser values of ξ produce error curves within those shown.

Investigation of $\sigma_{cp_c}(\gamma)$

For Class-A matrices, $\sigma_{cp_t}(\gamma) = 0$. The calculated cross section $\sigma_{cp_c}(\gamma)$ is zero onl' · ´ = 0 and $\pi/2$. The maximum calculation error occurs for imum value of $\sigma_{cp_c}(\gamma)$.

$$\sigma_{cp_{c\,max}}(\gamma) = \sigma_{cp_c}(\pi/4) = \frac{a_{11}^2}{2}\left[1 - \cos\xi\right] \qquad \text{(B-13)}$$

The maximum error is directly proportional to the element cross section and insensitive to the sign of the angular error for reasonable ξ . The following table relates maximum calculation error to the element cross section.

Table B-2 INVESTIGATIONS OF σ_{cp_c} - CLASS A MATRICES

$\xi°$	$\frac{\sigma_{cp_c}}{a_{11}^2}$ max - db
± 2	-32
± 5	-27
± 10	-23

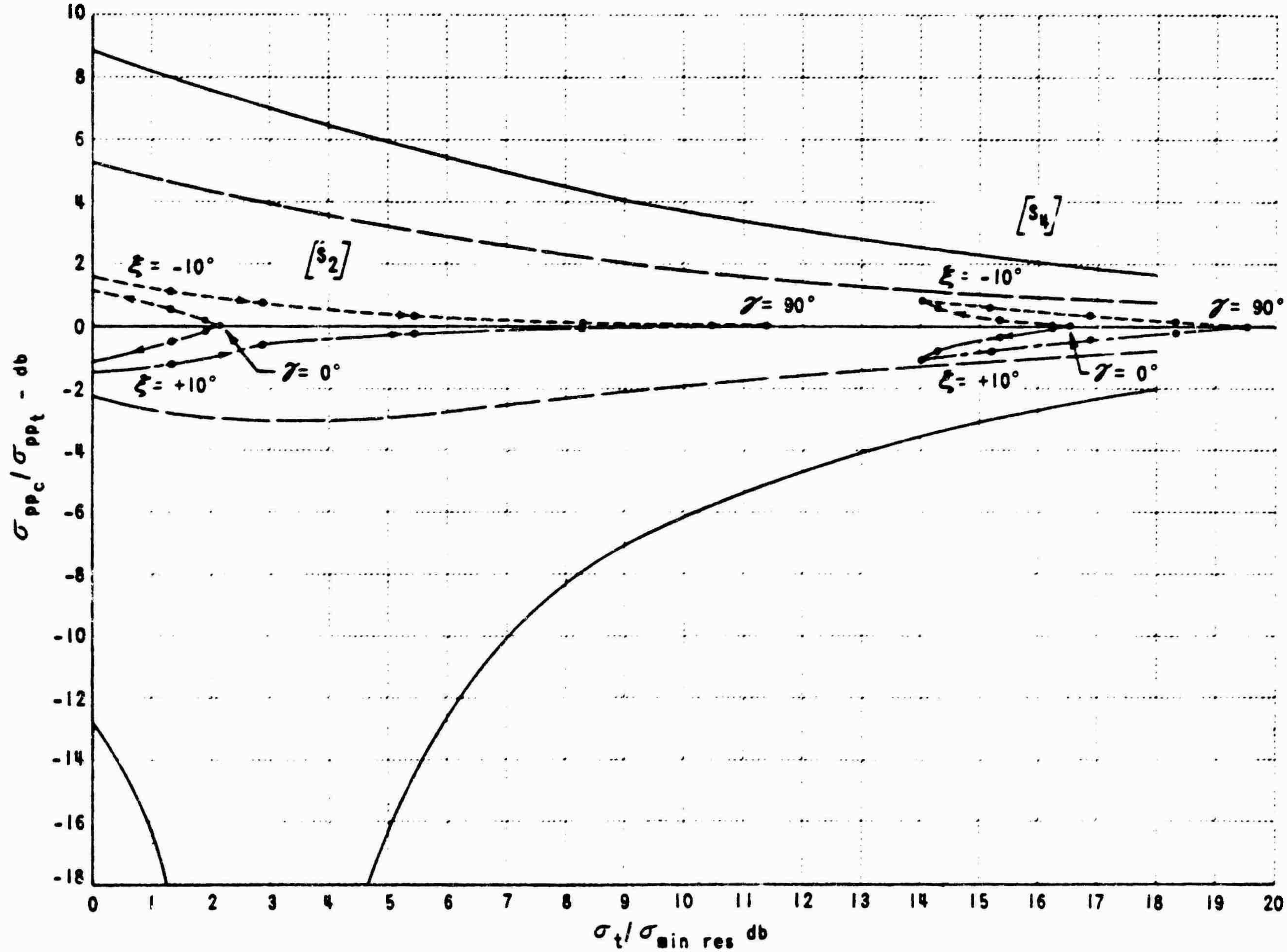
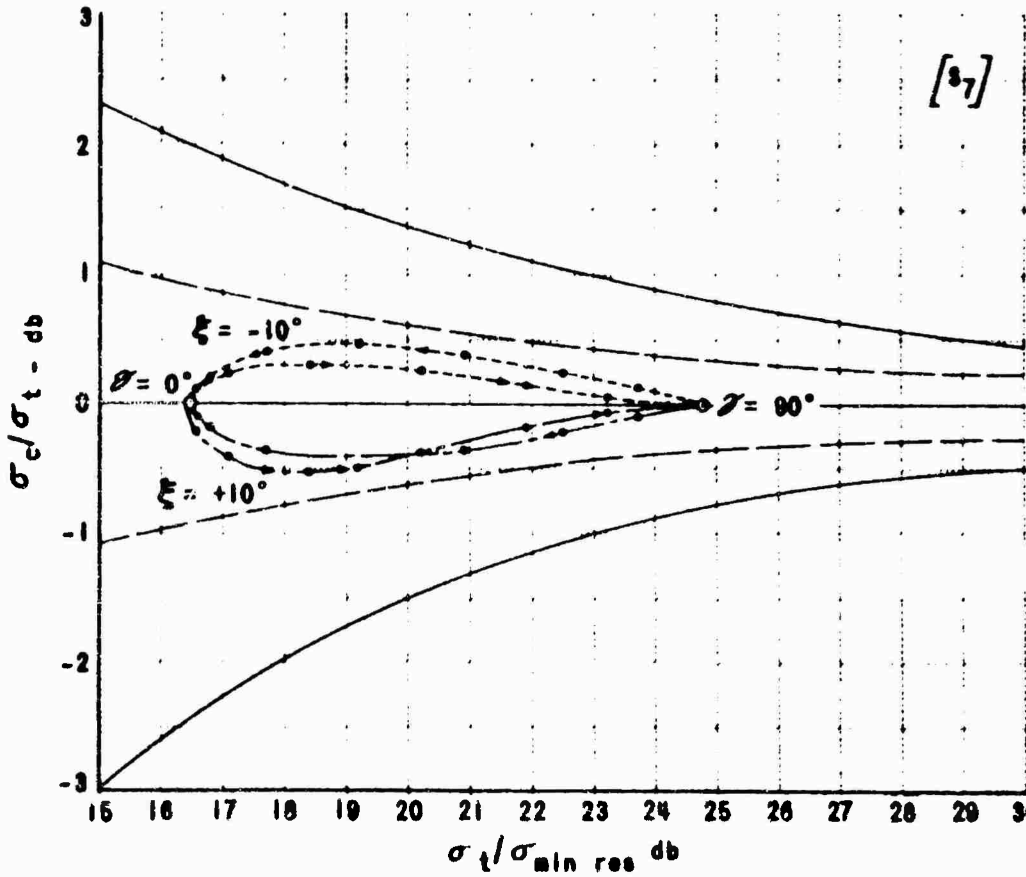


Figure B-9 PROGRAM 2 - ELEMENT PHASE ANGLE ERRORS, $\sigma_{pp_c}(\gamma)$ INVESTIGATION, CLASS B



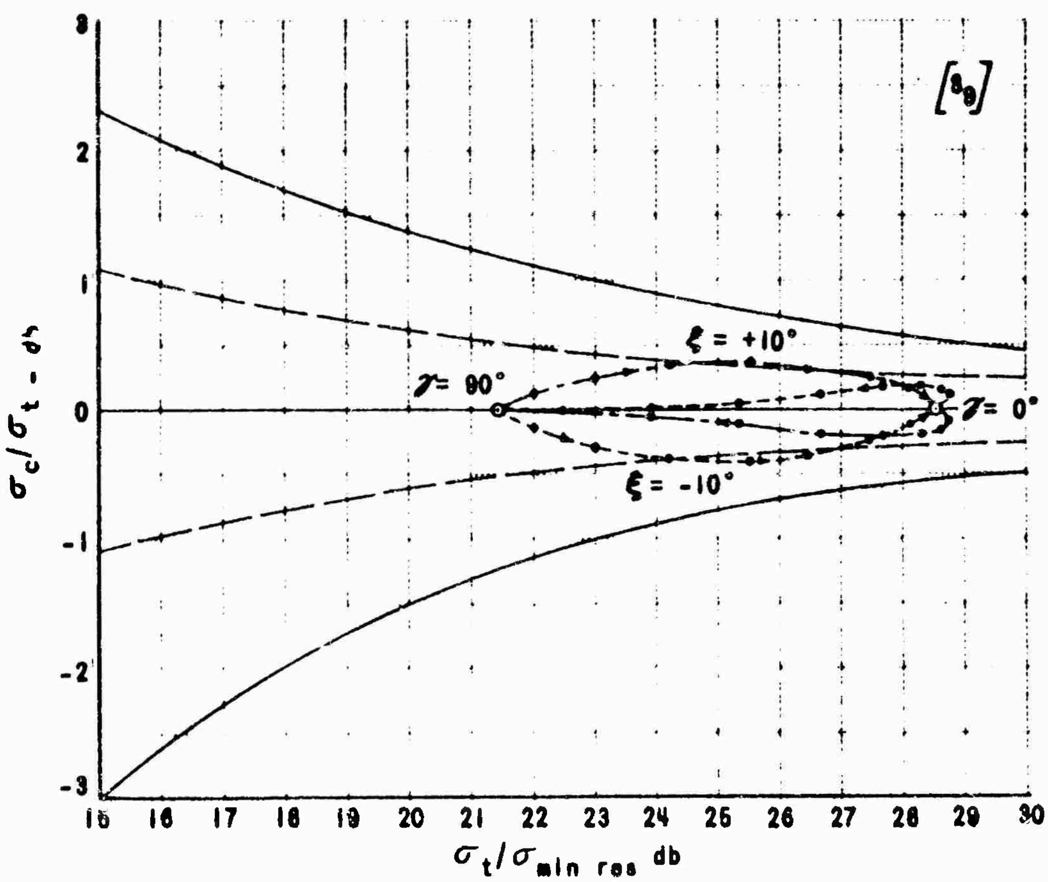


Figure B-10 PROGRAM 2 - ELEMENT PHASE ANGLE ERRORS, $\sigma_{pp_c}(\gamma)$ INVESTIGATION, CLASS C

Whereas the true cross section is zero, calculated cross sections will be appreciable for large a_{11}^2 and ξ. Five-degree measurement errors are considered tolerable. Class-B matrices generate errors according to the following relation:

$$\sigma_{cp_c}(\gamma) = a_{11}^2 \cos^4\gamma + a_{22}^2 \sin^4\gamma - 2a_{11}a_{22}\cos^2\gamma \sin^2\gamma \cos(\psi_s + \xi) \qquad \text{(B-14)}$$

The calculated error is zero for $\gamma = 0$ and $\pi/2$ and is maximum for $\gamma = \pi/4$.

$$\left(\frac{\sigma_{cp_c}}{\sigma_{cp_t}}\right)_{max} = \frac{a_{11}^2 + a_{22}^2 - 2a_{11}a_{22}\cos(\psi_s + \xi)}{a_{11}^2 + a_{22}^2 - 2a_{11}a_{22}\cos(\psi_s)} \qquad \text{(B-15)}$$

The above form is equivalent to Equation B-12 for unrestricted values of ψ_s. Matrices 2 and 4 produce small errors in calculated cross section. Figure B-11 shows the error curve of $\xi = \pm 10$ degrees. Matrix 3 gives rise to larger errors. Phase-angle determinations to ± 5 degrees are specified to hold the normalized cross-section inaccuracy to less than 3 db. The error curves given are straight lines because the error is insensitive to γ for $10 \leq \gamma \leq 80$ degrees. Class-C matrices give rise to nominal error curves for matrices 7 and 9 as shown in Figure B-12. Smaller ξ produce less calculation error. However, matrix 8 exhibits unacceptable errors for all phase angle errors studied. Figure B-13 contains the data on this sensitive matrix. It is seen that reduction in the angular error from ± 10 degrees to ± 5 degrees does not result in greater cross section accuracy as indicated in all other cases.

It is instructive to examine matrix 8 in detail. Because $a_{12}^2 < a_{11}^2$, $a_{12}^2 < a_{22}^2$, the important terms here are ($a_{11}^2 + a_{22}^2 - 2a_{11}a_{22}\cos(\psi_s + \xi)$). Now $a_{11}^2 = a_{22}^2$ and ψ_{st} = 6 degrees which is within the angular error range to produce $\psi_{sm} = 0$. The error value $\xi = -5$ degrees comes nearest to this situation and this angular error produces the maximum error in calculated cross section. None of the other representative matrices studied exhibit these large errors at any of the common cross sections computed. However, this critical situation is not limited to the $\sigma_{cp_c}(\gamma)$ case; it is easy to construct hypothetical matrices which would produce large calculation errors for small angular errors in any of the calculated cross section.

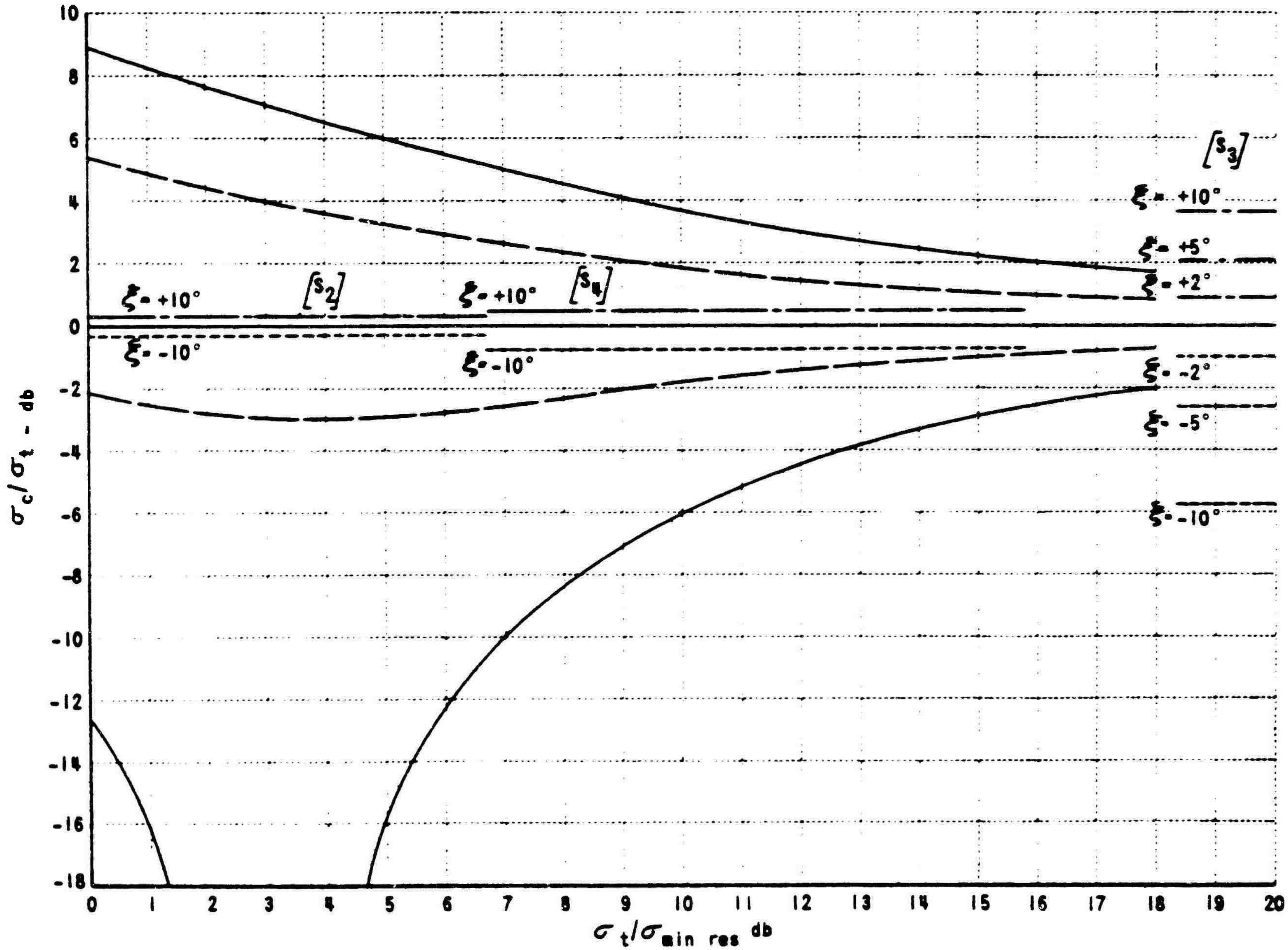
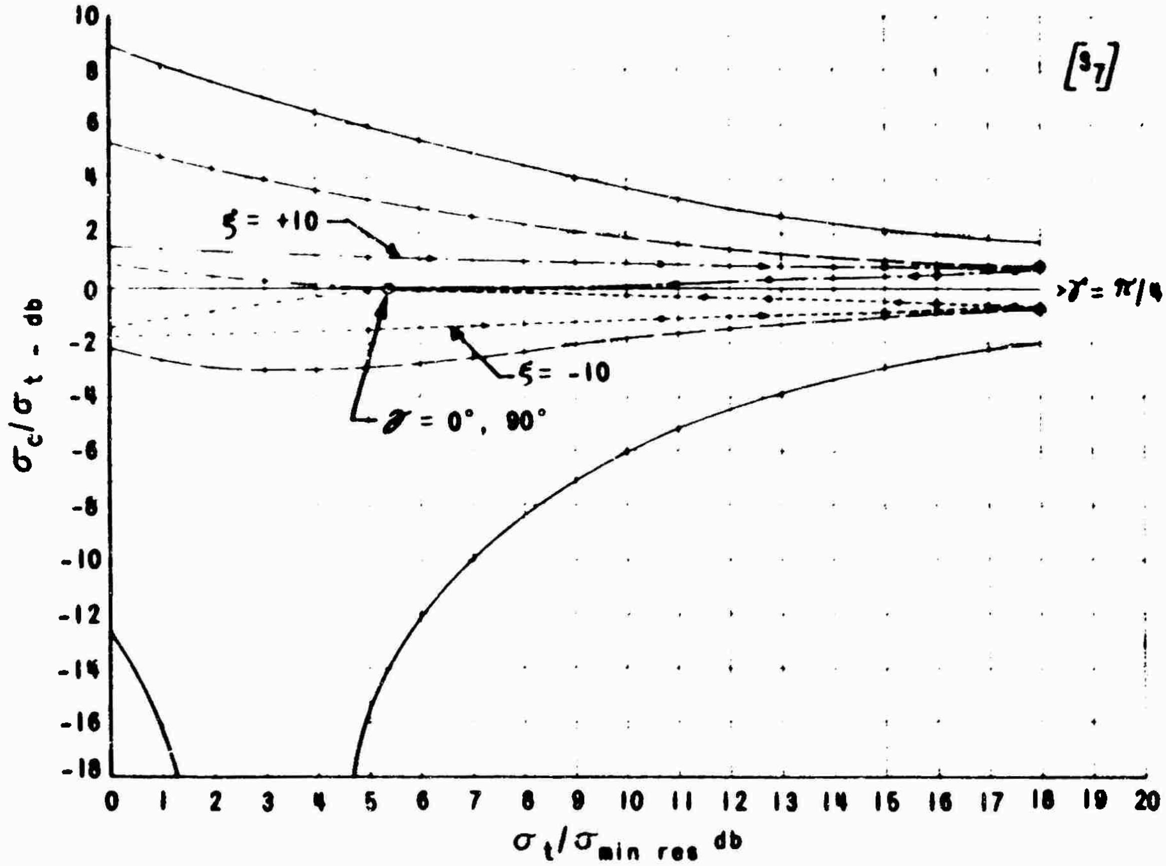


Figure B-11 PROGRAM 2 - ELEMENT PHASE ANGLE ERRORS, $\sigma_{cp_c}(\gamma)$ INVESTIGATION, CLASS B



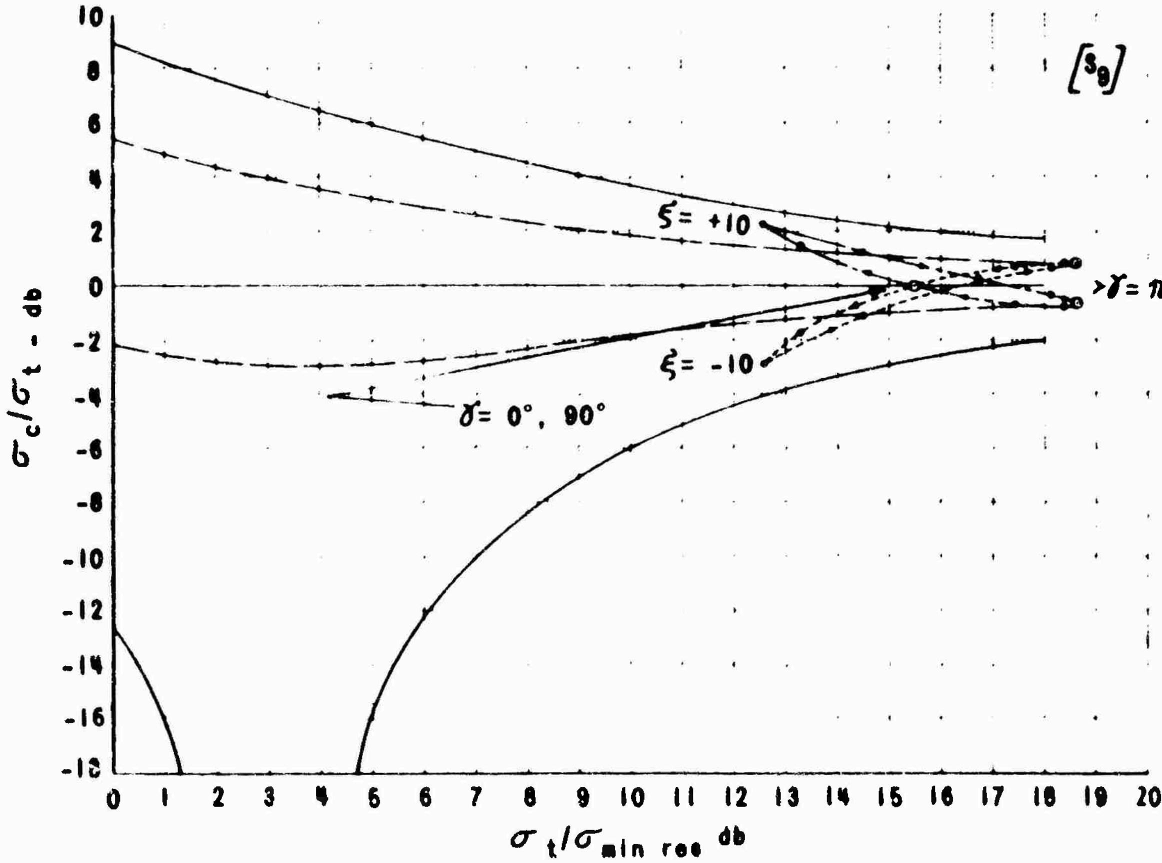


Figure B-12 PROGRAM 2 - ELEMENT PHASE ANGLE ERROR, $\sigma_{cp_c}(\gamma)$ INVESTIGATION, CLASS C

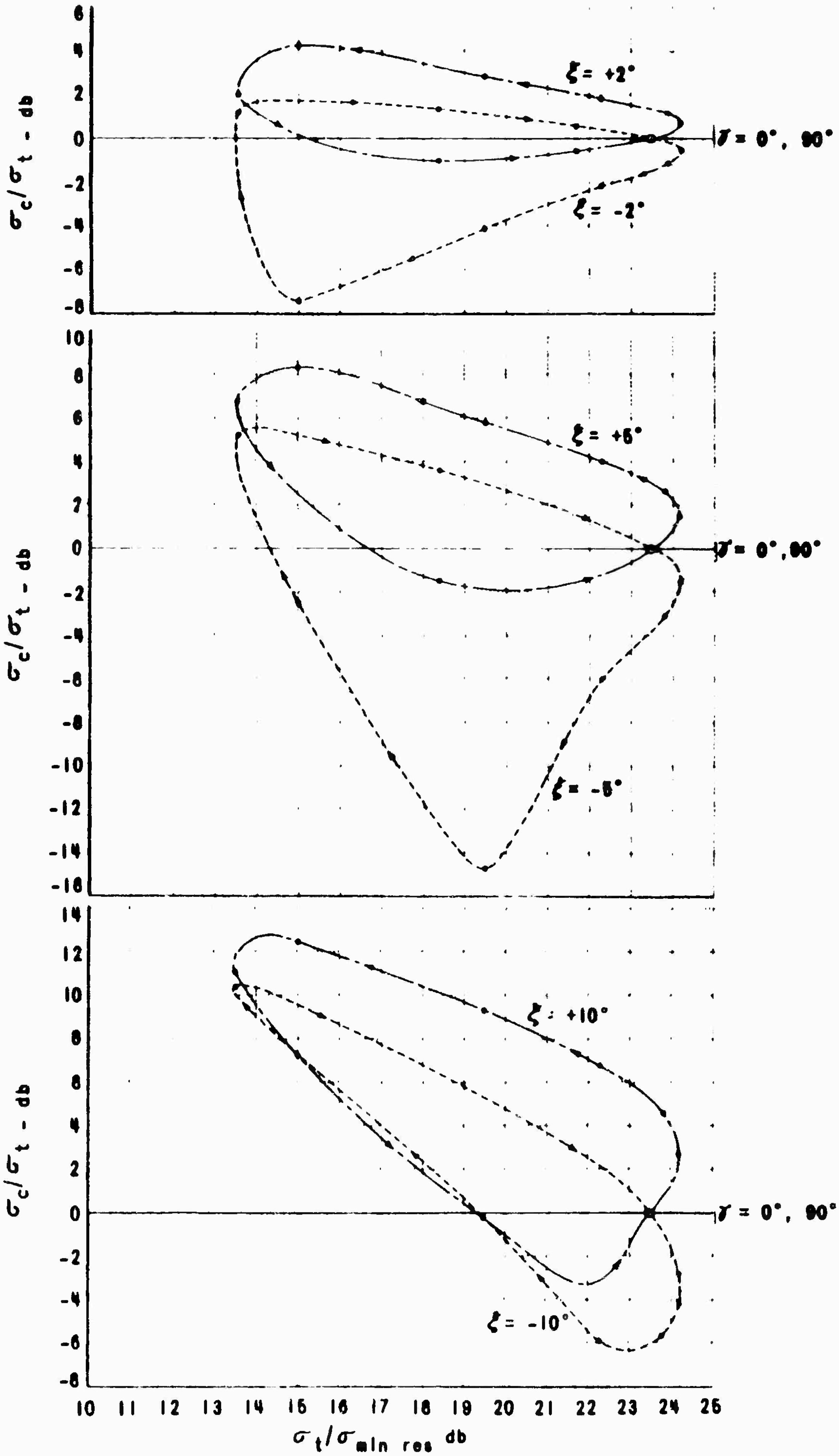


Figure B-13 PROGRAM 2 - ELEMENT PHASE ANGLE ERRORS, $\sigma_{cp_c}(\gamma)$ INVESTIGATION, CLASS C - MATRIX NO. 8

Investigation of σ_{RL_c}

Results need not be presented in this study. Class-A matrices produce errors in calculated cross section identical to those given in the Class-A study of σ_{pp_c} $(\gamma = \pi/4)$. Also, Class-B results equal Class-C results for σ_{RL_c} since a_{12} does not enter into this circular-polarization formulation. These errors are identical to those given in the σ_{pp_c} $(\gamma = \pi/4)$ analysis of Class-B matrices. The earlier results indicate small errors in calculated cross section for all matrices.

Investigation of σ_{RR_c}, σ_{LL_c}

Class-A matrices result in $\sigma_{RR_c} = \sigma_{LL_c} = \sigma_{cp_c}$ $(\gamma = \pi/4)$. Earlier analysis indicates that angular measurement errors should be held to ± 5 degrees or less.

Figure B-14 contains error points for Class-B matrices where $\sigma_{RR_c} = \sigma_{LL_c}$. Matrix 3 errors should be held to ± 5 degrees to maintain calculated cross sections within 3 db of true cross sections. This figure also shows errors associated with Class-C matrices. Again calculated cross sections based upon matrix 8 are most sensitive to angular errors. This study indicates a limit of ± 5 degrees on phase-angle measurement accuracy.

PROGRAM 3. ERRORS IN ELEMENT MAGNITUDE AND ELEMENT PHASE ANGLE

Both magnitude and phase-angle measurement errors were used to generate a measured scattering matrix from the true matrix parameters listed in Table B-1. In this study the contribution of background and noise to measurement error is reduced. Equation B-2 is replaced by

$$\sigma_m = \left[\left(\sqrt{\sigma_t} \pm \sqrt{\sigma_b/2}\right)^2 + \sigma_N/2\right] \qquad \text{(B-16)}$$

to simulate more closely average rather than worst-case measurement errors. The background interference, whether constructive or destructive, is arbitrarily handled in the following manner: use plus sign for all $a_{11_m}^2$, use minus sign for $a_{12_m}^2$ and $a_{22_m}^2$. This approach allows more realistic background simulation.

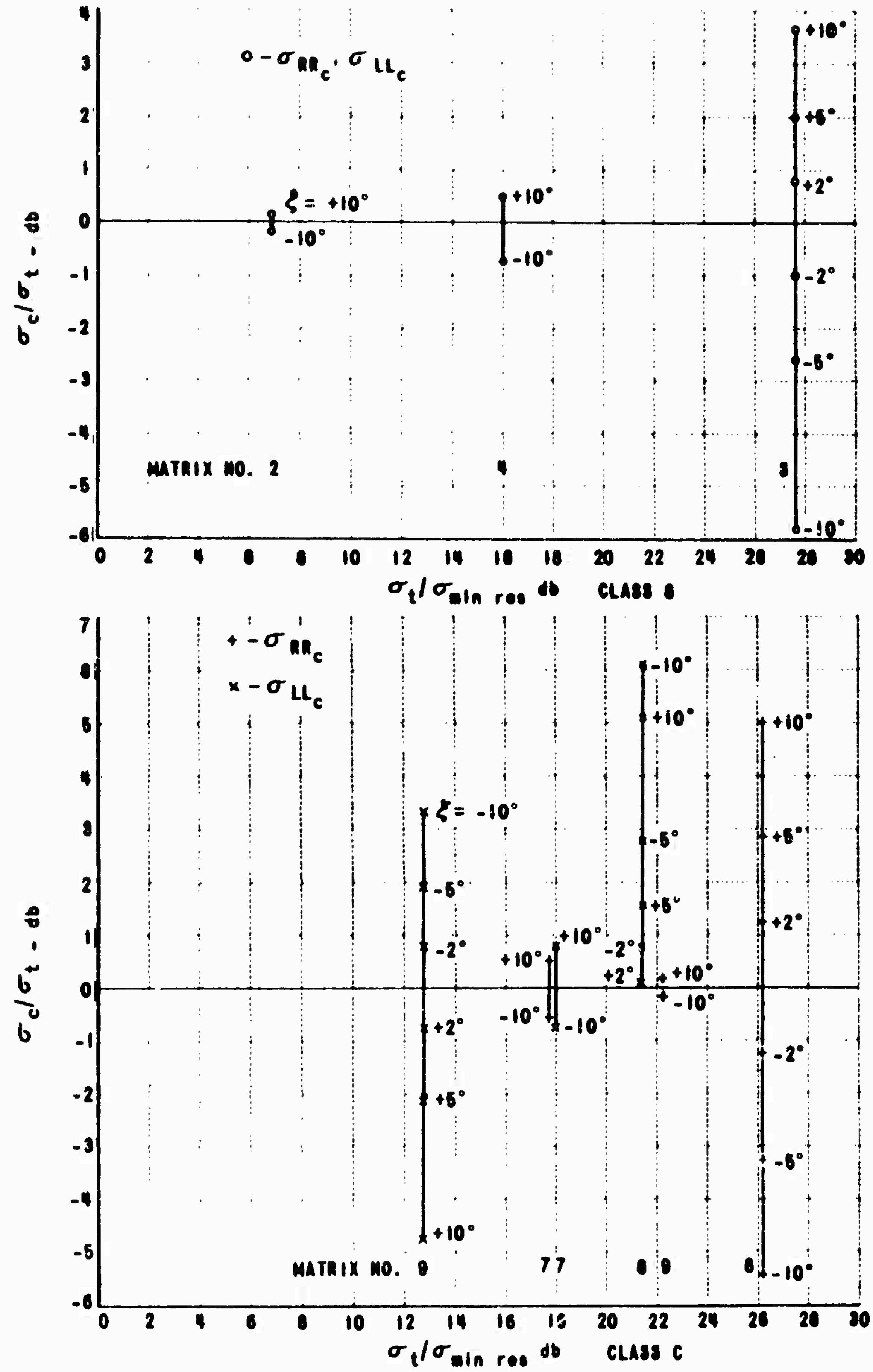


Figure B-14 PROGRAM 2 - ELEMENT PHASE ANGLE ERROR, σ_{RR_c}, σ_{LL_c} INVESTIGATION

Moderate phase angle errors are used in this study. All θ_s determinations are assumed to be 5 degrees too large and all ψ_s values to have errors of -5 degrees. A further program change is the restriction on $a_{12_m}^2$ for $a_{12}^2 = 0$: if $a_{12}^2 < \sigma_{min\ res}$ then $a_{12_m}^2 = 0$ in matrix formulation. This procedure was recommended following analysis of Program 1 data.

Investigation of $\sigma_{pp_c}(\gamma)$

Results for Class-A matrices are not plotted. The maximum cross-section error occurs at $\gamma = 0$ and $\pi/2$. The errors at these antenna orientations are simply the original element-magnitude measurement errors.

Class-B and Class-C error plots are given in Figures B-15 and B-16. Error curves associated with matrices 3 and 8 are not shown; the normalized errors are less than ± 0.1 db. All calculated cross sections are acceptable as a function of the level of the true cross section.

Investigation of $\sigma_{cp_c}(\gamma)$

For Class-A matrices $\sigma_{cp_t}(\gamma) = 0$. The calculated cross section is greatest for $\gamma = \pi/4$.

$$\sigma_{cp_{c\,max}} = \frac{1}{4}\left[a_{11_m}^2 + a_{22_m}^2 - 2a_{11_m}a_{22_m}\cos(\xi)\right] \quad \text{(B-17)}$$

Results are plotted in Table B-3. Matrix 1 gives the greatest error

Table B-3 INVESTIGATION OF $\sigma_{cp_{c\,max}}$, CLASS A

Matrix No.	$a_{11_t}^2$	$\frac{\sigma_{cp_{c\,max}}}{a_{11_t}^2}$
1	1.79×10^{-3} sq.m.	-15.5 db
5	2.68×10^{-1} sq.m.	-27.0 db
6	1.79×10^{-2} sq.m.	-23.4 db
10	1.13×10^{-1} sq.m.	-26.4 db

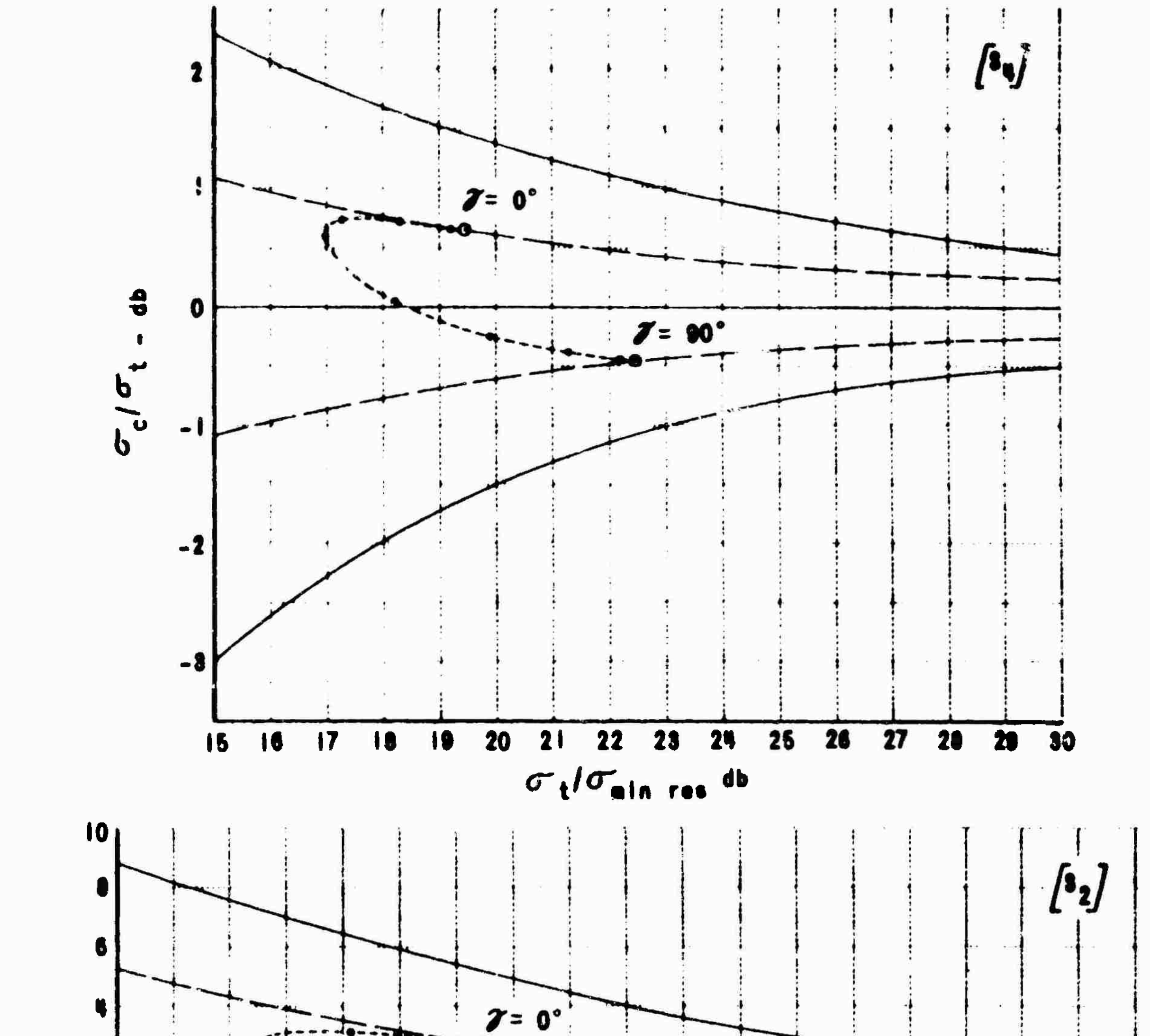


Figure B-15 PROGRAM 3 - MAGNITUDE AND PHASE ANGLE ERRORS, $\sigma_{pp_c}(\gamma)$ INVESTIGATION CLASS B

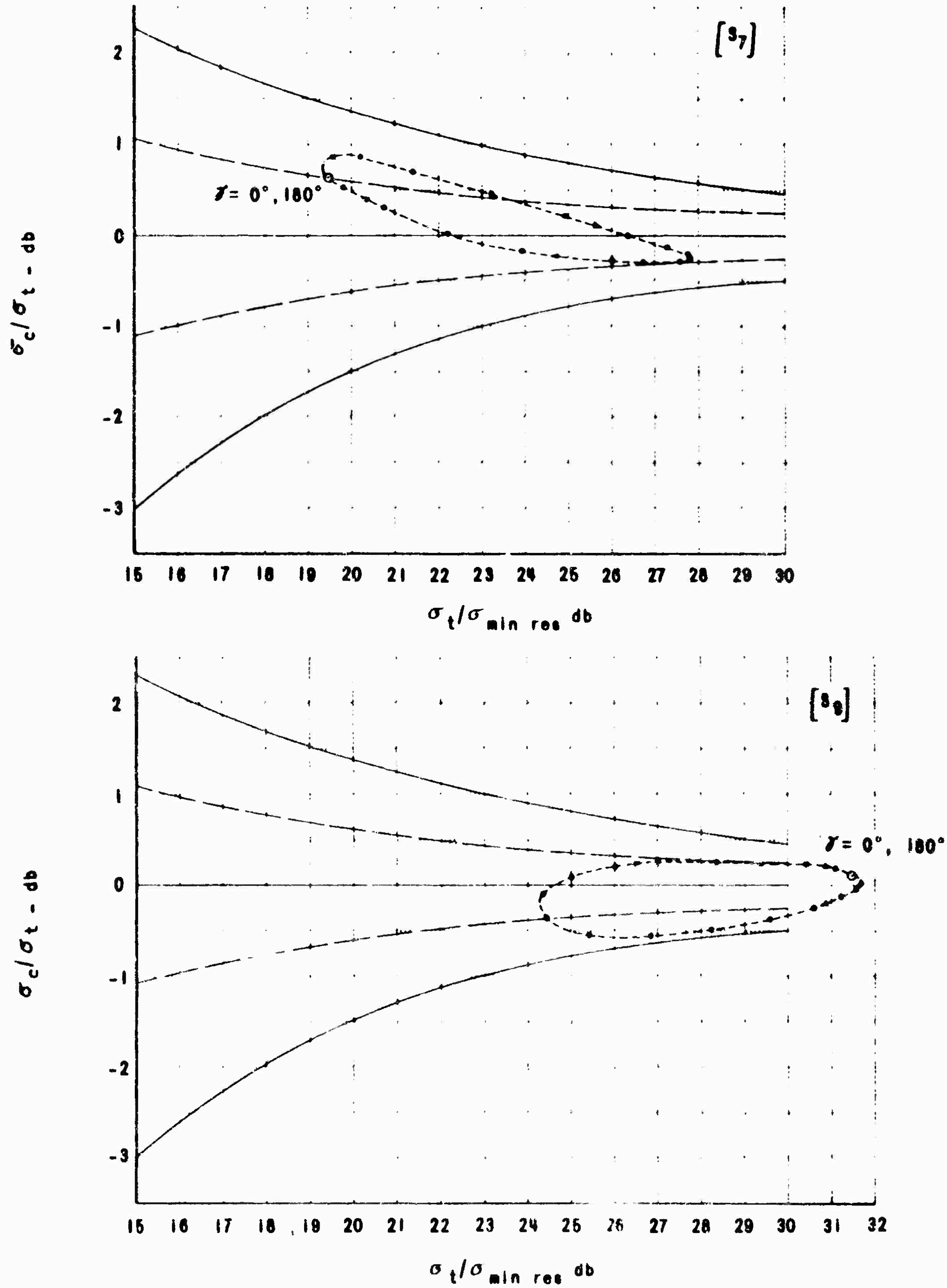


Figure B-16 PROGRAM 3 - MAGNITUDE AND PHASE ANGLE ERRORS, $\sigma_{pp_c}(\gamma)$ INVESTIGATION, CLASS C in normalized cross section, because magnitude measurement errors are greatest for this matrix. The particular manner in which magnitude measurement errors have been selected provides $a_{11_m} a_{22_m} = a_{11} a_{22}$ since signal level is sufficient to put the measurements in the symmetrical portion of the error curve. However, $(a_{11_m}^2 + a_{22_m}^2)$ shows maximum deviation from $(a_{11}^2 + a_{22}^2)$ at the lowest signal levels, i.e., for matrix-1 calculations.

Class-B matrices produce acceptable errors as shown in Figure B-17. The calculated and true cross sections are both zero for $\gamma = 0$ and $\pi/2$.

Class-C matrices give acceptable error curves for matrices 7 and 9. Matrix 8 provides the error curve shown in Figure B-18. At least 20 db discrepancy can exist between calculated and true cross section for σ_{cp_c} and this particular matrix. The ξ chosen for ψ_S allows parameter interactions discussed in the corresponding investigation of this matrix in Program 2.

Investigation of σ_{RL_c}

All matrices provide error points within the original measurement-error-bound curves. The results for σ_{RL_c} thus remain acceptable throughout all three programs. Figure B-19 shows the error points for this study.

Investigation of σ_{RR_c}, σ_{LL_c}

Figure B-19 also contains data on same-sense circular-polarization cross sections. Calculated cross sections are well behaved except in the case of matrices 3 and 8. These latter examples produce error points lying outside the maximum error bound curves generated by magnitude measurements alone (Figure B-2). Results are considered marginally acceptable.

SUMMARY AND CONCLUSIONS

The propagation of measurement errors in the use of scattering matrices has been the subject of a limited investigation. Three programs introducing measurement errors in element magnitudes --in element phase angles, and in the combination of these two matrix-parameters--were studied by calculating commonly desired

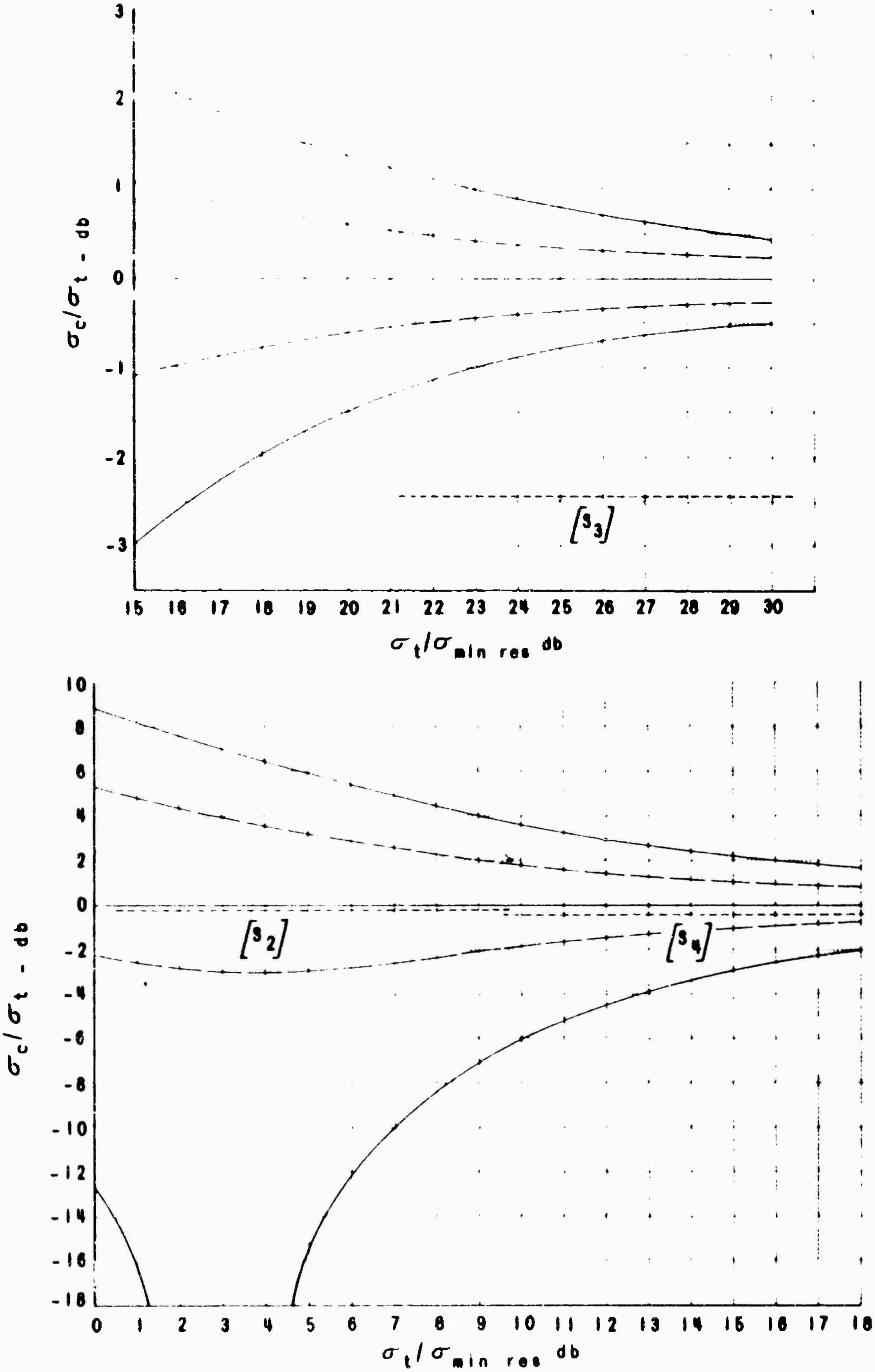


Figure B-17 PROGRAM 3 - MAGNITUDE AND PHASE ANGLE ERRORS, $\sigma_{cp_c}(\gamma)$ INVESTIGATION, CLASS B

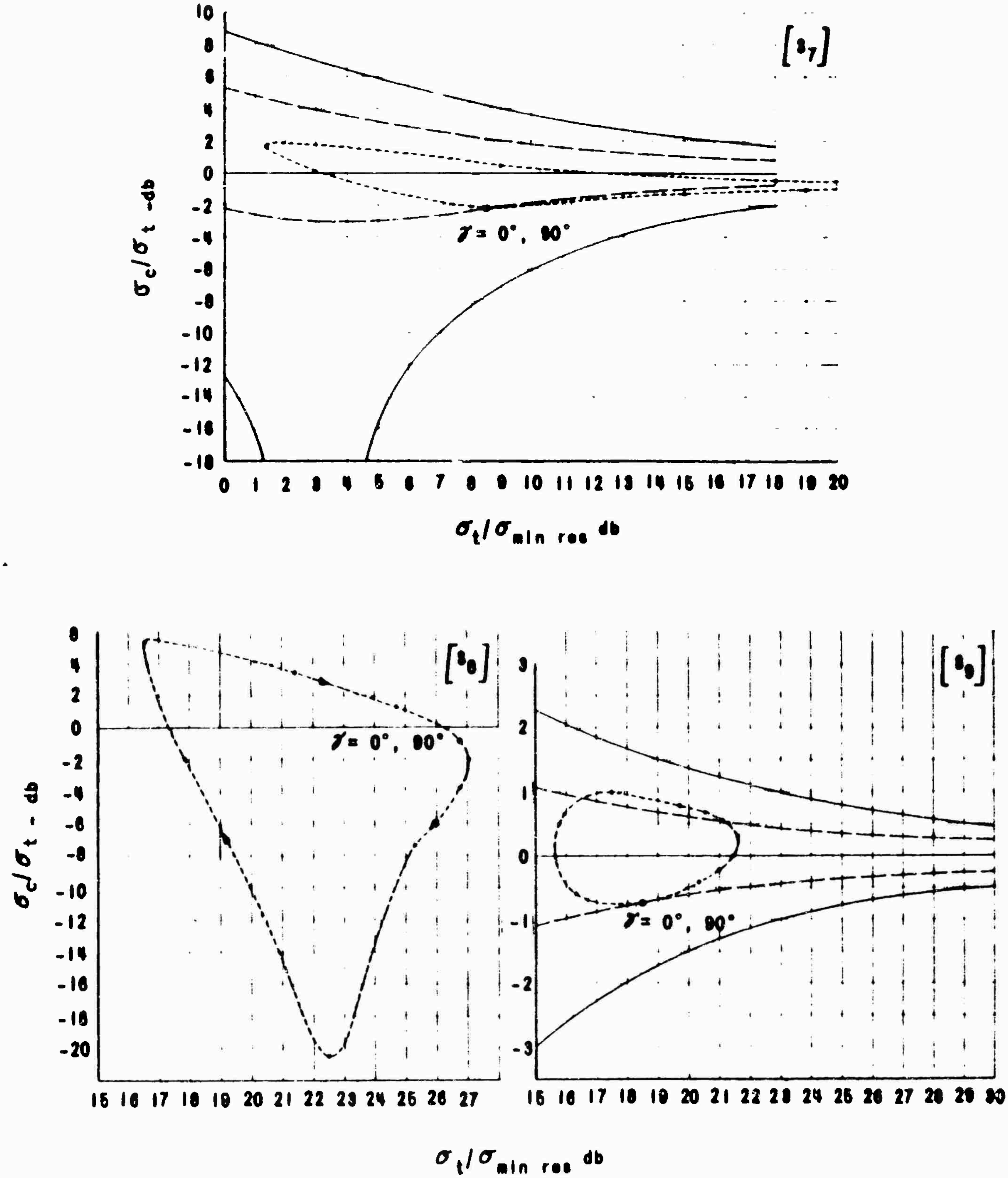
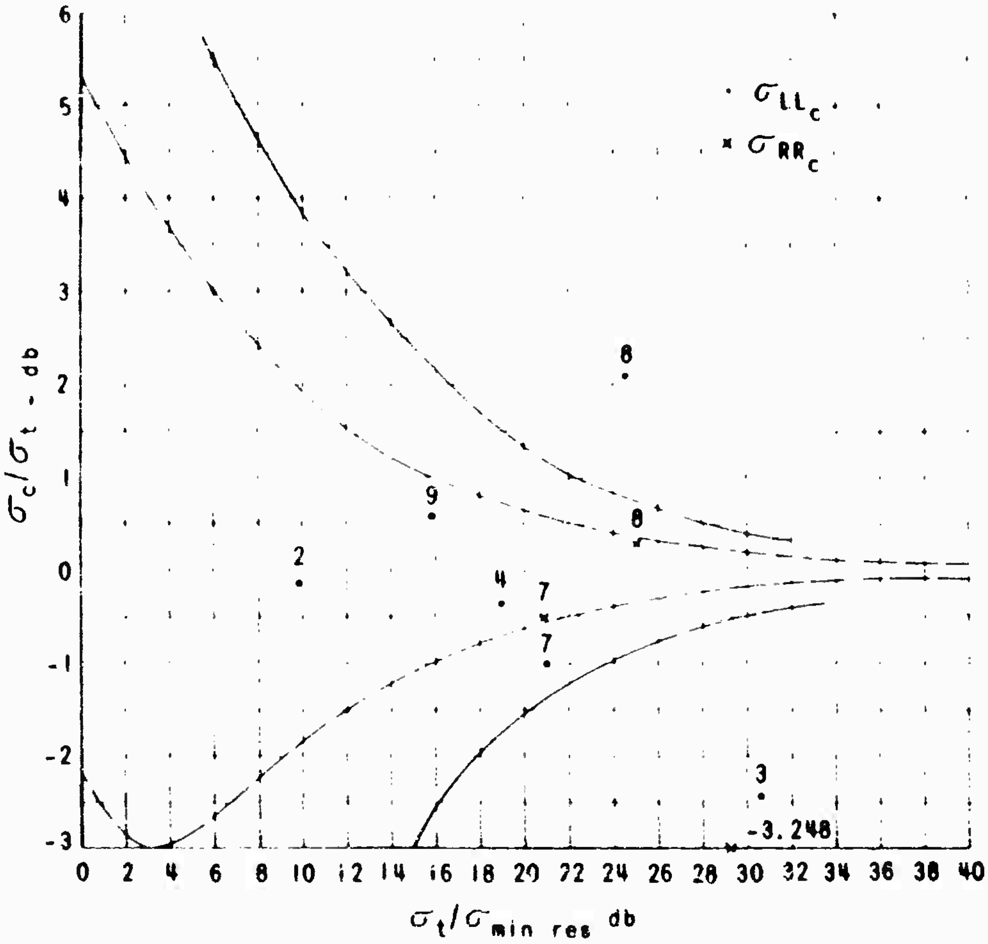


Figure B-18 PROGRAM 3 - MAGNITUDE AND PHASE ANGLE ERRORS, $\sigma_{cp_C}(\gamma)$ INVESTIGATION, CLASS C



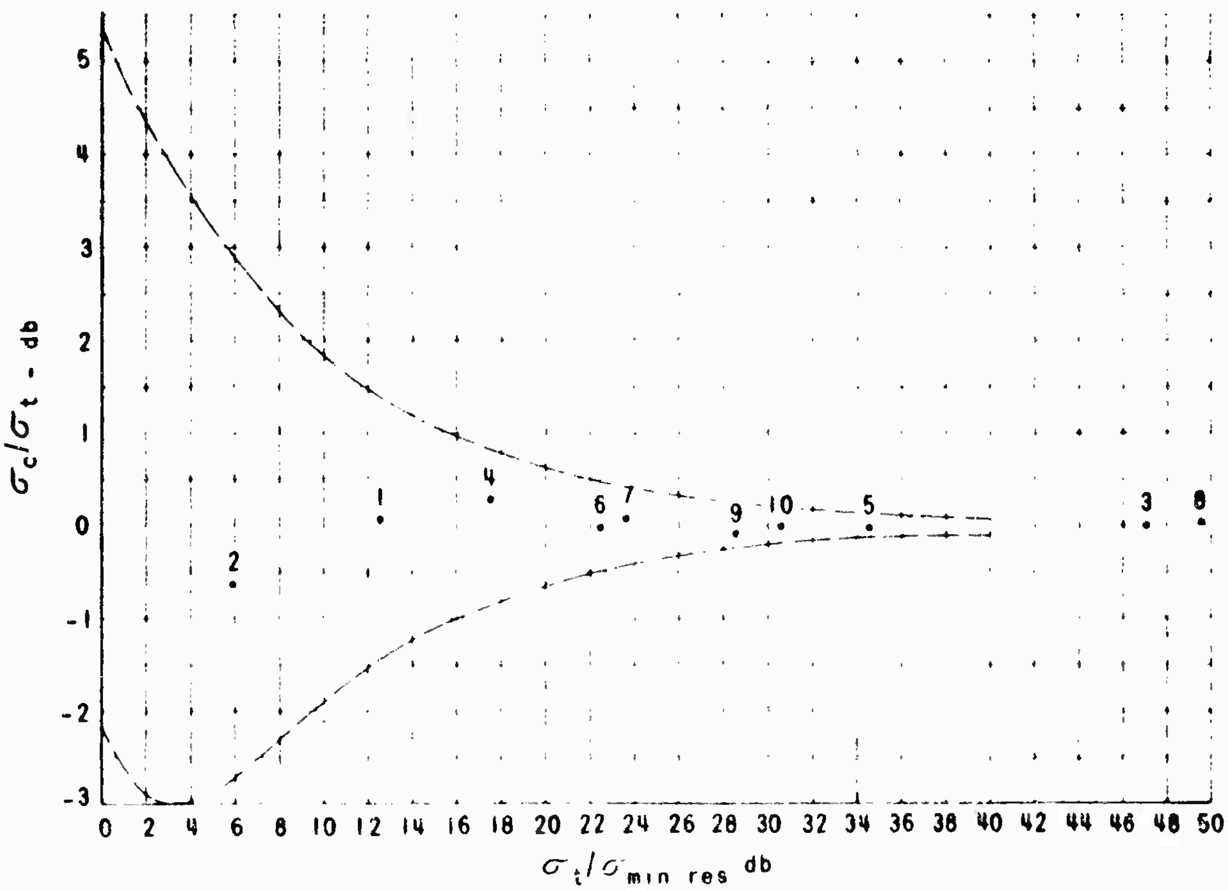


Figure B-19 PROGRAM 3 - MAGNITUDE AND PHASE ANGLE ERRORS, σ_{RL_c}, σ_{LL_c}, AND σ_{RR_c} INVESTIGATION, CLASSES A, B AND C cross sections from a set of ten tepresentative scattering matrices.

Program 1 treated errors inthe measurement of scattering-matrix element magnitudes. True matrix parameters are listed in Table B-1. Measured values of matrix-element magnitudes were generated according to error curves shown in Figure B-1. It was found that these measurement error curves represent a bound on the calculated cross section $\sigma_{R\ell_c}$, associated with opposite-sense circularly polarized antennas.

The calculation-error curves generated by $\sigma_{pp_c}(\gamma)_{max}$ (the maximum value of cross section calculated in the case of parallel linearly polarized antennas) for Class-A matrices (matrices describing specular reflection) are given in Figure B-2. These calculation-error curves represent an outer bound within which all other calculated cross sections lie.

When the true cross section is much larger than the minimum resolvable cross section (> 22 db), the error in the ratio of calculated cross section to true cross section is less than ± 1 db. Smaller values of true cross section make it desirable to reduce the outer error bound on calculated cross sections to the original measurement-error bound. This reduction can be accomplished by placing a limit on the minimum value of element magnitude a_{12_m} to be inserted in matrices having zero off-diagonal elements. In the present analysis a restriction of the form $a_{12_m}^2 = 0$ if $a_{12_t}^2 \leq \sigma_{min\ res}$ will reduce the outer bound to the original measurement-error bound. In this case all inaccuracies in calculated cross sections become acceptable. The effects of long-term gain variations may be included in this analysis by adding the gain change (in db) to the calculation errors listed in Figures B-1 to B-8.

Program 2 concerned errors in the measurement of element phase angles. Element magnitudes were assumed correct throughout this program. In general, it was found that ± 10-degree errors in the measurement of matrix phase angles produced inaccuracies in calculated cross section comparable to those observed in Program 1. In such cases, a decrease in phase-angle error resulted in less error in calculated cross section. In particular, scattering matrices can exist which produce unacceptable inaccuracies (> 10 db) in calculated cross sections for specific antenna-polarization combinations. The calculation of $\sigma_{cp_c}(\gamma)$ from matrix 8 illustrates this point. Matrix 8 also generates large errors in σ_{RR_c} and σ_{LL_c} . These cross-section inaccuracies are not necessarily reduced by a partial reduction in the phase-angle measurement error.

It is concluded that large errors in the determination of matrix phase angles ($\delta = \pm 10$ degrees) will result in acceptable calculation errors in almost all cases. Certain matrix forms will exist which produce unacceptable inaccuracies in the calculation of specific cross sections, even for small phase-angle measurement errors. Such difficulties could be recognized by examining the form of measured matrices prior to cross-section calculations.

Program 3 investigated measurement errors in both element magnitudes and element phase angles. The more realistic disposition of measurement errors does not alter the conclusions derived from Programs 1 and 2. Unacceptable inaccuracies in calculated cross sections again obtain for one particular matrix (matrix 8) and orthogonal-polarization cross sections $\sigma_{CP_c}(\gamma)$, σ_{RR_c} and σ_{LL_c} .

Errors in the measurement of matrix phase angles are the principal contributions to cross section inaccuracies in this instance. It is possible to establish criteria by which the susceptibility of a particular matrix to such errors can be estimated. All other matrices studied provide acceptable calculated cross sections for each of the commonly desired polarization combinations studied.

Finally, it should be noted here that large inaccuracies in calculated cross sections are a result of the scattering matrix form rather than the particular method by which matrix phase angle estimates are obtained. In order to limit the allowable number of these undesirable scattering matrices, matrix phase angles, in particular the parameter ψ_{s_m}, should be obtained as accurately as possible.

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13 ABSTRACT

The concept of the polarization scattering matrix is presented in this report. Measurement techniques leading to the determination of the scattering matrix on a radar range are discussed. The polarization dependence of measurements on a ground plane range are noted. Matrix-determination techniques are examined in the context of ground-plane-range operation. The utility of performing scattering matrix measurements on the RAT SCAT range is discussed. Finally a specific measurement technique is chosen for the scattering matrix solution; consideration of operational ease and measurement accuracy forms the basis of this choice. This report is No. 4 in a series of 8 RAT SCAT Research and Development Program Reports.

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